1 If the points (1,0,0), (0,3,0) and (0,0,2) lie on a plane, then the unit normal vector $\hat{n}$ to the plane is

(A) $\frac{1}{\sqrt{14}}(\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$

(B) $\frac{1}{7}(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

(C) $\frac{1}{\sqrt{14}}(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$

(D) $\frac{1}{7}(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$

(E) $\frac{1}{7}(6\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$

Ans: (E)

$$\frac{x}{1} + \frac{y}{3} + \frac{z}{2} = 1$$

$$6x + 2y + 3z = 6$$

$$\vec{N} = 6\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\hat{N} = \frac{6\hat{\imath} + 2\hat{\jmath} + 3\hat{k}}{\sqrt{36 + 4 + 9}} = \frac{1}{7}\{6\hat{\imath} + 2\hat{\jmath} + 3k\}$$

2 The equation of the plane through the point $(1, -5,3)$ and having a normal vector $\vec{n} = 2\hat{\imath} - 2\hat{\jmath} - \hat{k}$ is

(A) $2x + 2y + z = 9$

(B) $2x - 2y - z = 11$

(C) $2x + 2y - z = 9$

(D) $2x - 2y - z = 9$

(E) $2x - 2y - z = 13$

Ans: (D)

$$P(1, -5,3)$$

$$\vec{n} < 2 - 2 - 1 >$$

$$\Rightarrow 2(x - 1) + -2(y + 5) + -1(z - 3) = 0$$

$$2x - 2 - 2y - 10 - z + 3 = 0$$

$$2x - 2y - z - 9 = 0$$

3 If θ is angle between the lines $\frac{x}{1}=\frac{y+1}{2}=\frac{z-1}{3}$ and $\frac{x+1}{3}=\frac{y}{2}=\frac{z}{1}$, then $\cos\ \theta =$

(A) $\frac{5}{9}$

(B) $\frac{5}{8}$

(C) $\frac{5}{6}$

(D) $\frac{5}{7}$

(E) $\frac{6}{7}$

Ans: (D)

$$\cos\ \theta = \frac{\overrightarrow{b_1}\cdot\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$$

$$\cos\ \theta = \frac{1\times 3+2\times 2+3\times 1}{\sqrt{1^2+2^2+3^2}\sqrt{3^2+2^2+1^2}} = \frac{3+4+3}{\sqrt{14}\sqrt{14}}$$

$$= \frac{10}{14} = 5/7$$

4 The distance from the point (2,2,2) to the plane $2x - y + 3z = 5$ is equal to

(A) $\frac{3\sqrt{7}}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{3\sqrt{14}}{7}$

(D) $\frac{3\sqrt{14}}{14}$

(E) $\frac{\sqrt{3}}{3}$

Ans: (D)

$$d = \frac{|4-2+6-5|}{\sqrt{2^2+1^2+3^2}} = \frac{3}{\sqrt{4+1+9}} = \frac{3}{\sqrt{14}}\times\frac{\sqrt{14}}{\sqrt{14}} = \frac{3\sqrt{14}}{14}$$

5 The angle between the planes $x=\sqrt{3}$ and $z=\sqrt{2}$ is equal to

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

(E) 0

Ans: (D)

$\frac{\pi}{2}$

6. Three fair dice are rolled simultaneously. Let a, b, c be the numbers on the top of the dice. Then the probability that $\min(a, b, c) = 6$ is

(A) $\frac{1}{216}$

(B) $\frac{1}{36}$

(C) $\frac{1}{6}$

(D) $\frac{11}{216}$

(E) $\frac{5}{6}$

Ans: (A)

1/216

7. If A and B are two events such that $P(A) = 0.5, P(B) = 0.4$ and $P(A \cap B) = 0.2$, then $P(A \mid (A \cup B))$ is equal to

(A) $\frac{6}{7}$

(B) $\frac{5}{6}$

(C) $\frac{5}{7}$

(D) $\frac{4}{7}$

(E) $\frac{1}{2}$

Ans: (C)

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A/A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)}$$

$$= \frac{P(A)}{P(A) + R(B) - P(A \cap B)} = \frac{0.5}{0.5 + 0.4 - 0.2}$$

$$= \frac{0.5}{0.7} = 5/7$$

8. There are 37 men and 33 women at a party. If a prize is given to one person chosen at random, then the probability that the prize goes to a woman is

(A) $\frac{33}{70}$

(B) $\frac{32}{70}$

(C) $\frac{33}{80}$

(D) $\frac{37}{70}$

(E) $\frac{37}{80}$

Ans: (A)

$$\frac{33c_1}{70c_1} = \frac{33}{70}$$

9. A fair coin is tossed twice. Given that the first toss resulted in head, then the probability that the second toss also, would result in head is

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{3}{8}$

(D) $\frac{1}{2}$

(E) $\frac{5}{8}$

Ans: (D)

$$\frac{1}{2}$$

10. The coefficient of variation (C.V.) and the mean of a distribution are respectively 75 and 44. Then the standard deviation of the distribution is

(A) 30

(B) 31

(C) 32

(D) 33

(E) 35

Ans: (D)

$$cV = \frac{\sigma}{\bar{x}} \times 100$$

$$75 = \frac{\sigma}{44} \times 100$$

$$\sigma = \frac{75 \times 44}{100}$$

$$= 33$$

11. There are 4 red, 3 blue and 3 yellow marbles in an urn. If three marbles are drawn simultaneously, then the probability that the number of yellow marbles will be less than 2 is equal to

(A) $\frac{97}{120}$

(B) $\frac{49}{60}$

(C) $\frac{47}{60}$

(D) $\frac{59}{60}$

(E) $\frac{39}{60}$

Ans: (B)

12. In a box there are four marbles and each of them is marked with distinct number from the set {1,2,5,10}. If one marble is randomly selected four times with replacement and the number on it noted, then the probability that the sum of numbers equals 18 is

(A) $\frac{1}{64}$

(B) $\frac{3}{16}$

(C) $\frac{5}{32}$

(D) $\frac{3}{32}$

(E) $\frac{1}{32}$

Ans: (D)

13. $\lim_{t\to 0}\left(\frac{(2t-3)(t-2)}{t}-\frac{3(t+2)}{t}\right)$ is equal to

(A) 10

(B) -10

(C) -7

(D) 7

(E) 5

Ans: (B)

$$\begin{aligned}\mathrm{Lt}_{t\to 0}\quad & (2t-3)\times 1+(t-2)2-3\\ = \quad & -3-4-3=-10\end{aligned}$$

14. If $f(x)=\begin{cases} x^2\sin\left(\frac{\pi}{6}x\right) & \text{for } x\le -3\\ x\cos\left(\frac{\pi}{3}x\right) & \text{for } x>-3\end{cases}$, then the value of $\lim_{x\to -3^+} f(x)$ is equal to

(A) 3

(B) -3

(C) 9

(D) -9

(E) 0

Ans: (A)

$$-3\cos\left(\frac{\pi}{3}x-3\right)$$

$= -3x - 1 = 3$

15. $\lim_{x\to 0} \frac{\log(1+x)+1-e^x}{4x^2-9x}$ is equal to

(A) $\frac{-1}{9}$

(B) $\frac{1}{9}$

(C) $\frac{-1}{18}$

(D) $\frac{1}{18}$

(E) 0

Ans: (E)

$$\mathrm{Lt}_{x\to 0} \frac{\frac{1}{1+x} - e^x}{8x-9} = 0$$

16. $\lim_{t\to 0} \frac{\sin(t^2)}{t\sin(5t)}$ is equal to

(A) 5

(B) 25

(C) $\frac{1}{25}$

(D) $\frac{1}{5}$

(E) 0

Ans: (D)

$$\mathrm{Lt}_{t\to 0} \frac{\frac{\sin\, \mathrm{l}t^2}{t^2}}{\frac{\sin\, 5t}{t}} = 1/5$$

17. Let $f(x) = \begin{cases} 3x+6, & \text{if } x \geq c \\ x^2-3x-1, & \text{if } x < c \end{cases}$, where $x \in \mathbb{R}$ and c is a constant. The values of c for which f is continuous on $\mathbb{R}$ are

(A) $-7,1$

(B) $1,3$

(C) $-1,7$

(D) $-1,6$

(E) $2, -3$

Ans: (C)

$3c + 6 = c^2 - 3c - 1$
$c^2 - 6c - 7 = 0$
$(c - 7)(c + 1) = 0$
$c = -1,7$

18. If $\lim_{x \to -2} \frac{3x^2+ax-2}{x^2-x-6}$ is a finite number, then the value of a is equal to

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

Ans: (D)

$$\text{Lt}_{x \to -2} \frac{12 - 2a - 2}{4 + 2 - 6} = \frac{12 - 2a - 2}{0}$$
$10 - 2a = 0$
$a = 5$

19. If $x = \sqrt{10^{\cos^{-1}\theta}}$ and $y = \sqrt{10^{\sin^{-1}\theta}}$, then $\frac{dy}{dx}$ is equal to

(A) xy

(B) $\frac{x}{y}$

(C) $\frac{y}{x}$

(D) $\frac{-x}{y}$

(E) $\frac{-y}{x}$

Ans: (E)

$xy = \sqrt{10}\pi/2$
$$\frac{dy}{dx} + y = 0$$
$$\frac{dy}{dx} = -y/x.$$

20. If $y = e^{3\log(2x+1)}$, then $\frac{dy}{dx} =$

(A) $6e^{3\log(2x+1)}$

(B) $6\dfrac{e^{3\log(2x+1)}}{2x+1}$

(C) $\dfrac{e^{3\log(2x+1)}}{2x+1}$

(D) $\frac{e^{3\log(2x+1)}}{3(2x+1)}$

(E) $(2x+1)e^{3\log(2x+1)}$

Ans: (B)

$$\frac{dy}{dx} = e^{3\log(2x+1)} \cdot \frac{3}{2x+1} \times 2$$
$$= 6\frac{e^{3\log(2x+1)}}{2x+1}$$

21. If $x\sin\ y + y\sin\ x = \pi$, then $\frac{dy}{dx}$ at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is equal to

(A) 1

(B) $\frac{\pi}{2}$

(C) -1

(D) $\dfrac{-\pi}{2}$

(E) 0

Ans: (C)

$$\sin\ y + x\cos\ y \cdot y' + y'\sin\ x + y\cos\ x = 0$$
$$1 + 0 + y' + 0 = 0$$
$$y' = -1$$

22. Let $f(x) = \begin{cases} \tan\ x, \text{ if } 0 \le x \le \frac{\pi}{4} \\ ax + b, \text{ if } \frac{\pi}{4} < x < \frac{\pi}{2} \end{cases}$ If $f(x)$ is differentiable at $x = \frac{\pi}{4}$, then the values of a and b are

respectively

(A) $2, \frac{2-\pi}{2}$

(B) $2, \frac{4-\pi}{4}$

(C) $1, \frac{-\pi}{4}$

(D) $2, \frac{-\pi}{4}$

(E) $22 \quad 2,1-\pi$

Ans: (A)

$$1 = a\frac{\pi}{4} + b$$
$$b = 1 - \frac{a\pi}{4} \quad 2 = a$$
$$b = 1 - \frac{\pi}{2} = \frac{2-\pi}{2}$$

23. $\frac{d}{dx}\left(\frac{1}{x}\frac{d^2}{dx^2}\left(\frac{1}{x^3}\right)\right) =$

(A) $-36x^{-7}$

(B) $36x^{-7}$

(C) $72x^{-6}$

(D) $72x^{-7}$

(E) $-72x^{-7}$

Ans: (E)

$$\frac{1}{x} \times 12x^{-5}$$
$$\Rightarrow \quad 12x^{-6}$$
$$\Rightarrow \quad -72x^{-7}$$

24. Air is blown into a spherical balloon. If its diameter d is increasing at the rate of 3 cm/min, then the rate at which the volume of the balloon is increasing when $d = 10$ cm, is

(A) $120\pi \text{cm}^3/\text{min}$

(B) $150\pi \text{cm}^3/\text{min}$

(C) $100\pi \text{cm}^3/\text{min}$

(D) $180\pi \text{cm}^3/\text{min}$

(E) $210\pi \text{cm}^3/\text{min}$

Ans: (B)

$$v = \frac{4}{3}\pi v^3$$
$$= \frac{4}{3}\pi \frac{d^3}{8} = \frac{\pi d^3}{3 \times 2}$$
$$\frac{dv}{dt} = \frac{\pi}{3 \times 2} \times 3d^2 \times \frac{d}{dt}(d)$$
$$= \frac{\pi \times 100 \times 3}{2}$$
$$= \frac{150\pi}{}$$

25. The equation of tangent to the circle $(x-5)^2 + y^2 = 25$ at $(2,4)$ is

(A) $3x - 4y + 10 = 0$

(B) $x + y = 6$

(C) $2x - y = 0$

(D) $3x - 2y + 2 = 0$

(E) $3x - 4y - 10 = 0$

Ans: (A)

$$(x-5)(x_1 - 5) + yy_1 = 25$$
$$(x, y,) = (2,4,$$
$$(x-5)x - 3 + 4y = 25$$
$$-3x + 15 + 4y = 25$$
$$-3x + 4y = 10$$
$$3x - 4y + 10 = 0$$

26. If x and y are both non-negative and if $x + y = \pi$, then the maximum value of $5\sin\ x\sin\ y$ is equal to

(A) 1

(B) $\sqrt{5}$

(C) 5

(D) -5

(E) 0

Ans: (C)

$$\begin{aligned} f(x) &= 5\sin\ x \cdot \sin\ y \\ f(x) &= 5\sin\ x\sin\ (\pi - x) \\ &= 5\sin\ x \cdot \sin\ x \\ f(x) &= 5\sin^2\ x \\ f'(x) &= 0\ \sin\ 2x = 0 \end{aligned}$$

$$\begin{aligned} f'(x) = 0 \quad & \sin\ 2x = 0 \\ x \quad & = \pi/2 \\ & \Rightarrow 5 \end{aligned}$$

27. The normal to the curve $y = \sqrt{x}$ at the point (25,5) intersects the y-axis at

(A) (0,245)

(B) (0,255)

(C) (255,0)

(D) (245,0)

(E) (0,100)

Ans: (B)

$$y' = \frac{1}{2\sqrt{x}}$$

$$m = \frac{1}{2\sqrt{25}} = \frac{1}{10}$$

$$(y - 5) = -10(x - 25)$$

$$y - 5 = -10$$

$$y = 250$$

(0,255)

28. The function $f(x) = x^5 e^{-x}$ is increasing in the interval

(A) $(5, \infty)$

(B) $(4, \infty)$

(C) $(-4, \infty)$

(D) $(-\infty, 5)$

(E) $(-5, \infty)$

Ans: (D)

$f'(x) = 5x^4 e^{-x} x^5 e^{-x}$

$= e^{-x} x^4 (5 - x)$

$(-\infty, 5)$

29. If $x + 13y = 40$ is normal to the curve $y = 5x^2 + \alpha x + \beta$ at the point (1,3), then the value of $\alpha\beta$ is equal to

(A) 15

(B) -6

(C) 6

(D) 13

(E) -15

Ans: (E)

$m = \frac{-1}{13}$

$y' = 10x + \alpha$
$m = 10 + \alpha$
$10 + \alpha = 13$
$\alpha = 3$

$3 = 5 + \alpha + \beta$
$3 = 5 + 3 + \beta$
$\beta = -5$

$\alpha_\beta = -15$

30. Let $f(x) = \cos\ x$ for $0 \leq x \leq \frac{\pi}{3}$. Then the value of c which satisfies the conclusion of the Mean Value Theorem for the function f on $\left[0, \frac{\pi}{3}\right]$ is equal to

(A) $\sin^{-1}\left(\frac{3}{2\pi}\right)$

(B) $\sin^{-1}\left(\frac{1}{3\pi}\right)$

(C) $\sin^{-1}\left(\frac{\pi}{12}\right)$

(D) $\sin^{-1}\left(\frac{1}{6\pi}\right)$

(E) $\sin^{-1}\left(\frac{\pi}{4}\right)$

Ans: (A)

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$
$$-\sin\ c = \frac{1/2 - 1}{\pi/3} = \frac{-3}{2\pi}$$
$$f'(x) = -\sin\ x$$

31. $\int \frac{e^{\frac{1}{\sqrt{t}}}}{t\sqrt{t}} dt =$

(A) $\frac{1}{2} e^{\frac{1}{\sqrt{t}}} + C$

(B) $\frac{-1}{2} e^{\frac{1}{\sqrt{t}}} + C$

(C) $2e^{\frac{1}{\sqrt{t}}} + C$

(D) $-2e^{\frac{1}{\sqrt{t}}} + C$

(E) $e^{\frac{1}{\sqrt{t}}} + C$

Ans: (C)

Put $e^{1/\sqrt{t}} = y$

$$\frac{d}{dt}\left(\frac{1}{\sqrt{t}}\right) = \frac{d}{dt}\left(t^{-1/2}\right)$$
$$= \frac{-1}{2}t^{-3/2}$$
$$\Rightarrow \frac{-1}{2} \times \frac{1}{t\sqrt{t}} dtdy$$
$$= \frac{1}{t\sqrt{t}} dt = -2dy$$

32. $\int \frac{\sin^{25} x}{\cos^{27} x} dx$ is equal to

(A) $\frac{\sin^{26}(x)}{26} + C$

(B) $\frac{\cos^{26}(x)}{26} + C$

(C) $\tan^{26}(x) + C$

(D) $\frac{\tan^{26}(x)}{26} + C$

(E) $26\tan^{26}(x) + C$

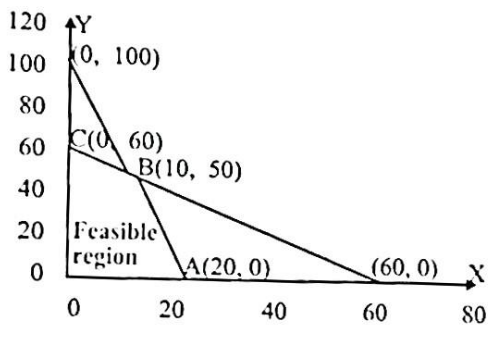
Ans: (D)

$\int \tan^{25} x \cdot \sec^2 x dx$

$\tan x = t$
$\sec^2 x dx = dt$

$$\int t^{25} dt = \frac{t^{26}}{26} + c$$
$$= \frac{\tan^{26}(x)}{26} + c.$$

33. The feasible region for a L.P.P. is shown in the figure below. Let $z = 50x + 15y$ be the objective function, then the maximum value of z is



(A) 900

(B) 1000

(C) 1250

(D) 1300

(E) 1520

Ans: (C)

1250

34. $\int \frac{1}{x^3}\sqrt{1-\frac{1}{x^2}}\,dx =$

(A) $\frac{-1}{6}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

(B) $\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

(C) $\frac{-1}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

(D) $\frac{4}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

(E) $\frac{-4}{3}\left(1-\frac{1}{x^2}\right)^{\frac{3}{2}}+C$

Ans: (D)

Put $1-\frac{1}{x^2}=t$

$\frac{1}{x^3}dx = 2\cdot dt$

$\int \sqrt{t}\cdot 2dt = 2\int \sqrt{t}dt$

$= 2\left(\frac{t^{3/2}}{3/2}\right)+c$

35. $\int (\tan^2(2x)-\cot^2(2x))dx =$

(A) $\frac{-1}{2}(\tan\ 2x + \cot\ 2x) + C$

(B) $2(\tan\ 2x + \cot\ 2x) + C$

(C) $\frac{1}{2}(\tan\ 2x - \cot\ 2x) + C$

(D) $\frac{-1}{2}(\tan\ 2x - \cot\ 2x) + C$

(E) $\frac{1}{2}(\tan\ 2x + \cot\ 2x) + C$

Ans: (E)

$\int (\sec^2 (2x) - 1) - (\text{cosec}^2\ 2x - 1)dx$
$\int [\sec^2 (2x) - 1 - \text{cosec}^2 (2x) + 1]dx$

$\frac{\tan\ 2x}{2} + \frac{\cot\ 2x}{2} + c$

$\frac{1}{2}(\tan\ 2x + \cot\ 2x) + c$

36. $\int \sin^3\ x dx + \int \cos^2\ x \sin\ x dx =$

(A) $-\cos\ x + C$

(B) $-\sin\ x + C$

(C) $x - \cos\ x + C$

(D) $x - \sin\ x + C$

(E) $\cos\ x - \sin\ x + C$

Ans: (A)

$\int \sin^3 x dx + \int \cos^2 x \sin x dx -$
$\int \sin x(\sin^2 x + \cos^2 x)dx$
$$\int \quad \sin x = -\cos x + c$$

37. $\displaystyle\int \frac{dx}{x^2 - x} =$

(A) $\log \frac{|x|}{|x-1|} + C$

(B) $\frac{-1}{x^2} + \log |x-1| + C$

(C) $x\log |x-1| + C$

(D) $\log \frac{|x-1|}{|x|} + C$

(E) $-x\log |x-1| + C$

Ans: (D)

$\int \frac{dx}{x(x-1)} \quad \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$

$1 = A(x-1) + Bx$
$1 = A(O-1) \Rightarrow -A = 1$

Put $x = 1$

$1 = B \; B = 1$

$\frac{A}{-} = -1$

$= \quad -\log |x| + \log |x-1|$
$= \quad \log \left|\frac{x-1}{x}\right| + C$

38. The value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cot x}{\sin x} dx$ is equal to

(A) $\frac{-1}{2}$

(B) $\frac{1}{2}$

(C) $\frac{-3}{2}$

(D) $\frac{3}{2}$

(E) 1

Ans: (E)

$\int_{\pi/6}^{\pi/2} \frac{\cos\ x}{\sin^2\ x} dx \qquad \sin\ x = t$

$\cos\ x dx = dt$

$\int \frac{dt}{t^2}$

$\int_{-1/2}^{1} t^{-2} dt = \left[\frac{t^{-1}}{-1}\right]_{1/2}^{1} = -(1-2)$

39. The area bounded by the curve $y = x(2 - x)$ and the line $y = x$ is

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{5}{6}$

(E) $\frac{2}{3}$

Ans: (A)

$$
\begin{aligned}
y &= 2x - x^2 \\
-y &= x^2 - 2x \\
&= (x-1)^2 + 1 \\
-y + 1 &= (x-1)^2 \\
-(y-1) &= (x-1)^2
\end{aligned}
$$

40. The value of $\int_{-1}^{2} (x - 2|x|) dx$ is equal to

(A) $\frac{-1}{2}$

(B) $\frac{-3}{2}$

(C) $\frac{-5}{2}$

(D) $\frac{-7}{2}$

(E) $\frac{-9}{2}$

Ans: (D)

$\int_{-1}^{0}(x+2x)+\int_{0}^{2}(x-2x)dx$

$\int_{-1}^{0}3xdx+\int_{0}^{2}-xdx.$

41. The value of $\int_{-10}^{10}\frac{x^{10}\sin x}{\sqrt{1+x^{10}}}dx$ is equal to

(A) $\frac{1}{100}$

(B) $\frac{-1}{100}$

(C) $\frac{1}{50}$

(D) $\frac{-1}{50}$

(E) 0

Ans: (E)

42. If $f(x)=\begin{cases}\cos x & \text{for } x\geq 0\\ 2x & \text{for } x<0\end{cases}$, then the value of $\int_{-2}^{\frac{\pi}{2}}f(x)dx$ is equal to

(A) 2

(B) -2

(C) -3

(D) 3

(E) 0

Ans: (C)

$\int_{-2}^{\pi/2}f(x)dx\Big) = \int_{-2}^{0}2x \quad +\int_{0}^{\pi/2}\cos\ xdx.$

$x^2]_{-2}0+\sin\ x]_0^{\pi/2} \quad = -4+\sin\ \pi/2-\sin\ 6$

$= -4+1=-3$

43 The value of $\int_0^{\frac{\pi}{16}} \cos\ 6x \cos\ 2x dx$ is equal to

(A) $\dfrac{1+\sqrt{2}}{16}$

(B) $\dfrac{1+\sqrt{2}}{8}$

(C) $\dfrac{2+\sqrt{2}}{16}$

(D) $\dfrac{-1+\sqrt{2}}{16}$

(E) $\dfrac{-1+\sqrt{2}}{8}$

Ans: (A)

$$\cos\ 6x \cdot \cos\ 2x = \frac{1}{2}(\cos\ 8x + \cos\ 4x)$$

$$\frac{1}{2} \int_0^{\pi/16} (\cos\ 8x + \cos\ 4x) dx$$

$$\frac{1}{2}\left[\frac{\sin\ 8x}{8} + \frac{\sin\ 4x}{4}\right]_0^{\pi/16}$$

$$\frac{1}{2}\left[\frac{1}{8} + \frac{1/\sqrt{2}}{4}\right]^{\frac{1}{2}} \left(\frac{1}{8} + \frac{1}{4\sqrt{2}}\right) = \frac{1+\sqrt{2}}{16}.$$

44. A particular solution of the differential equation $\frac{dy}{dx} = xy^2$ with $y(0) = 1$ is

(A) $y = \dfrac{2-x^2}{2}$

(B) $y = \dfrac{2}{2-x^2}$

(C) $y = \dfrac{2}{x^2} - 2$

(D) $y = \dfrac{x^2-2}{2}$

(E) $y = \dfrac{2}{x^2-2}$

Ans: (B)

$$
\begin{aligned}
dy &= xy^2 dx \\
xdx &= \frac{1}{y^2} dy \\
-\frac{1}{y} &= \frac{x^2}{2} + c. \\
-1 &= 0 + c \\
c &= -1
\end{aligned}
$$

45. The general solution of the differential equation $(x^2y^2 + y)dx - (x - 2x^3y)dy = 0$ is

(A) $x^2y^2 - \frac{y}{x} = C$

(B) $x^3y + \frac{x}{y} = C$

(C) $xy^2 + \frac{y}{x} = C$

(D) $xy^2 - \frac{y}{x} = C$

(E) $x^2y + \frac{y}{x} = C$

Ans: (D)

$$
\begin{aligned}
&x^2y^2dx + ydx - xdy + 2x^3ydy = 0 \\
&x^2y^2dx + 2x^3ydy = xdy - ydx \\
&y^2dx + 2xydy = \frac{xdy - ydx}{x^2} \\
&y^2x = \frac{y}{x} + c.
\end{aligned}
$$

46. The integrating factor of the differential equation $4xdy - e^{-2y}dy + dx = 0$ is

(A) e^{-2y}

(B) e^{2x^2}

(C) e^{4y}

(D) e^{-4y}

(E) x^4

Ans: (C)

$$
\begin{aligned}
&4xdy - e^{-2y}dy + dx = 0 \\
&dy(4x - e^{-2y}) = -dx \\
&4x - e^{-2y} = \frac{-dx}{dy}
\end{aligned}
$$

$$\frac{dx}{dy} + 4x = e^{-2y} \ \frac{dx}{dy} = e^{-2y} 4x$$

47. Consider the linear programming problem:
Maximize $z = 10x + 5y$
subject to the constraints

$$2x + 3y \leq 120$$
$$2x + y \leq 60$$
$$x, y \geq 0$$

Then the coordinates of the corner points of the feasible region are

(A) (0,0), (30,0), (0,40) and (15,30)

(B) (0,0), (60,0), (0,40) and (15,30)

(C) (0,0), (30,0), (0,60) and (15,30)

(D) (0,0), (30,0), (0,40) and (30,40)

(E) (0,0), (60,0), (0,40) and (30,40)

Ans: (A)

$$2x + y = 60$$
$$2x + 3y = 120$$
$$\frac{x}{60} + \frac{y}{40} = 1$$

48. Let $A = \{1,2,3,4,5\}$ and let $B = \{1,2,3,4\}$. If the relation $R: A \to B$ is given by $(a, b) \in R$ if and only if $a + b$ is even, then $n(R)$ is equal to

(A) 10

(B) 16

(C) 20

(D) 12

(E) 6

Ans: (A)

$A: \{1,3,5\}$ $A: \{2,4\}$
$B: \{1,3\}$
$B: \{2,4\}$

$a + b$:even

49. The domain of the function $f(x) = (x^2 - 2x - 63)^{3/2}, x \in \mathbb{R}$ is

(A) $(-\infty, -6] \cup [9, \infty)$

(B) $(-\infty, -9] \cup (7, \infty)$

(C) $(-\infty, -7] \cup [7, \infty)$

(D) $(-\infty, -5] \cup [9, \infty)$

(E) $(-\infty, -7] \cup [9, \infty)$

Ans: (E)

$x^2 - 2x - 63 \geq 0$
$(x - 9)(x + 7) \geq 0$

50. Let $A = \{x \in \mathbb{Z}: -1 \leq x < 4\}$ and let $B = \left\{x \in \mathbb{Z}: 0 < \frac{x}{2} \leq 3\right\}$. Then $A \cap B$ is equal to

(A) $\{1,2,3\}$

(B) $\{2,3\}$

(C) $\{1,2,3,4\}$

(D) $\{2,3,4\}$

(E) $\{0,1,2,3\}$

Ans: (A)

$A = \{-1,0,1,2,3\}$
$B = \{1,2,3,4,5,6\}$
$$0 < \frac{x}{2} \leq 3$$
$0 < x \leq 6$
$A \cap B = \{1,2,3\}$

51. Let $f(x) = \begin{cases} x + 2, & \text{for } x < 1 \\ 4x - 1, & \text{for } 1 \leq x \leq 3 \\ x^2 + 5, & \text{for } x > 3 \end{cases}$. Then

(A) $f(x)$ is not continuous at $x = -1$

(B) $f(x)$ is continuous at $x = 1$

(C) $f(x)$ is continuous at $x = 3$

(D) $f(x)$ is not continuous at $x = 5$

(E) $f(x)$ is not continuous at $x = 2$

Ans: (B)

at $x = 1$
$\angle HL = 3$
RHL $= 3$
at $x = 3$
$\angle HL = 11$
$RHL = 14$

52. Let $\odot$ be a binary operation on $\mathbb{Q} - \{0\}$ defined by $a \odot b = \frac{a}{b}$. Then $1 \odot (2 \odot (3 \odot 4))$ is equal to

(A) $\frac{3}{2}$

(B) $\frac{8}{3}$

(C) $\frac{4}{3}$

(D) $\frac{3}{4}$

(E) $\frac{3}{8}$

Ans: (E)

$$3 \odot 4 = \frac{3}{4}$$
$$2 \odot \frac{3}{4} = \frac{2}{3/4} = 8/3$$
$$1 \odot \frac{8}{3} = 1/8/3 = 3/8$$

53. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \cos\ x$. Then

(A) f is one - one and odd

(B) f is odd but not one - one

(C) f is even and onto

(D) f is one - one and even

(E) f is even but not onto

Ans: (E)

not one-one

not onto

even

54. If $n(A \cup B) = 97, n(A \cap B) = 23$ and $n(A - B) = 39$, then $n(B)$ is equal to

(A) 52

(B) 55

(C) 58

(D) 62

(E) 65

Ans: (C)

$97 - (39 + 23)$
$97 - 62$
$= \underline{35}$
$23 + 35 = 58$

55. The principal argument of the complex number $z = \frac{8+4i}{1+3i}$ is equal to

(A) $\frac{\pi}{4}$

(B) $\frac{-\pi}{4}$

(C) $\frac{3\pi}{4}$

(D) $\frac{-3\pi}{4}$

(E) $\frac{\pi}{6}$

Ans: (B)

$$z = \frac{8+4i}{1+3i} \times \frac{1-3i}{1-3i}$$
$$= \frac{20-i20}{10} = 2-2i$$

56. The minimum value of $|z+1|+|z-2|$ is equal to

(A) 1

(B) 2

(C) 3

(D) 4

(E) 0

Ans: (C)

$$|z_1+z_2| \leq |z_1|+|z_2|$$
$$|z_1-z_2| \leq |z_1|+|z_2|$$
$$3 \leq |z+1|+|z-2|$$

57. If $z = \frac{(3+i)(7-i)^2}{3-i}$, then the value of $|z|$ is equal to

(A) 48

(B) $\sqrt{50}$

(C) 50

(D) $\sqrt{500}$

(E) $\sqrt{48}$

Ans: (C)

$$|z| = \frac{|3+i||7-i|^2}{|3-i|}$$
$$= (\sqrt{49+1})^2 = 50$$

58. The value of $\left[\frac{5i}{(1-i)(2-i)(3-i)}\right]^{50}$ is equal to

(A) $\left(\frac{1}{2}\right)^{25}$

(B) $\left(\frac{1}{2}\right)^{50}$

(C) $-\left(\frac{1}{2}\right)^{25}$

(D) $-\left(\frac{1}{2}\right)^{50}$

(E) $\left(\frac{1}{10}\right)^{50}$

Ans: (B)

$$\left\{5i \times \frac{1+i}{1^2+1^2} \times \frac{2+i}{2^2+1^2} \frac{\times 3+i}{3^2+1}\right\}^{50}$$
$$= \left\{\frac{5i(1+i)(2+i)(3+i)\}50}{2 \times 5 \times 10}\right\}^{50}$$
$$= \left\{\frac{i(1+3i)(3+i)}{20}\right\}^{50}$$
$$= \left\{\frac{i \times 10i}{20}\right\}^{50}$$

59. If $z^4 = 7 - 5i$, then Im $((\bar{z})^4)$ is equal to

(A) 5

(B) 7

(C) -7

(D) -5

(E) 0

Ans: (A)

$Z4 = 7 + 5''$

$\text{lm }\{7 + 5i\} = 5$

60. The modulus of $\left(\frac{1+i}{1-i}\right)^{75} - \left(\frac{1-i}{1+i}\right)^{75}$ is

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) 4

(E) 16

Ans: (B)

$$\frac{1+i}{1-i} = i \quad i^{75} - (-i)^{75}$$
$$= i^{75} + i^{75} = i^3 + i^3$$
$$= -i - i = -2i$$

61. If z_1 and z_2 are two different complex numbers with $|z_2| = 1$, then $\left|\frac{1-\bar{z}_1 z_2}{z_1 - z_2}\right|$ is equal to

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{4}$

(E) 1

Ans: (E)

$$\frac{(1-\overline{z_1}z_2)(1-z_1\overline{z_2})}{(z_1-z_2)(\overline{z_1}-\overline{z_2})}$$

62. If $-1 + 7i, -1 + xi$ and $3 + 3i$ are the three vertices of an isosceles triangle which is right angled at $-1 + xi$, then the value of x is equal to

(A) -1

(B) 3

(C) -3

(D) 7

(E) -7

Ans: (B)

63. The sum of the first 24 terms of the series $9 + 13 + 17 + \cdots$ is equal to

(A) 1212

(B) 1200

(C) 1440

(D) 1320

(E) 1230

Ans: (D)

$12[2 \times 9 + 23 \times 4]$

64. In an A.P. there are 18 terms and the last three terms of the A.P. are 67,72,77. Then the first term of the A.P. is

(A) -7

(B) 9

(C) -9

(D) -8

(E) 7

Ans: (D)

$a + 17 \times 5 = 77$

65. If the first term of a G.P. is 3 and the sum of second and third terms is 60 , then the common ratio of the G.P. is

(A) 4 or -3

(B) 4 only

(C) 4 or 5

(D) 4 or -5

(E) -5 only

Ans: (D)

$$3r + 3r^2 = 60$$
$$r + r^2 - 20 = 0$$
$$(r+5)(r-4) = 0$$
$$r = -5,4$$

66. If n^{th} term of a series is $n + (-1)^{n-1}, n = 1,2,3, \ldots$, then the sum of first 40 terms of the series is

(A) 810

(B) 820

(C) 821

(D) 819

(E) 780

Ans: (B)

$$\frac{40 \times 41}{2}$$

67. The 11^{th} term of the geometric series $\sum_{r=0}^{20} 2 \times (-2)^r$ is equal to

(A) -4096

(B) 1024

(C) 2048

(D) 1048

(E) -2024

Ans: (C)

$$2 \times (-2)^{10}$$

68. Let S_n be the sum of the first n terms of the series $a_1 + a_2 + \cdots a_n + \cdots$. If $S_n = n^2 + 4n$, then the n^{th} term a_n is

(A) $2n + 3$

(B) $2n - 1$

(C) $2n + 5$

(D) $2n - 3$

(E) $2n$

Ans: (A)

$$a_n = s_n - s_{n-1}$$
$$= n^2 + 4n - [(n-1)^2 + 4(n-1)]$$
$$= 2n + 3$$

69. Let $t_n = \frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2$ for $n = 1,2,3,\ldots$. Then t_{10} is equal to

(A) $\frac{7}{600}$

(B) $\frac{231}{100}$

(C) $\frac{209}{600}$

(D) $\frac{11}{200}$

(E) $\frac{77}{200}$

Ans: (B)

$$\frac{1}{n}\cdot\frac{1}{n}\sum_{k=1}^{n}k^2$$
$$= \frac{1}{n^2}n(n+1)(2n+1)$$
$$T_{10} = \frac{1}{100}\times 10 \times 11 \times 21$$

70 The number of arrangements containing all the seven letter of the word ALRIGHT that begins with LG

is

(A) 720

(B) 120

(C) 600

(D) 540

(E) 760

Ans: (B)

$ALRIGHT$

71 The number of numbers greater than 6000 that can be formed from the digits 3,5,6,7 and 9 (no digit is repeated in a number) is equal to

(A) 264

(B) 720

(C) 192

(D) 132

(E) 544

Ans: (C)

$12 \times 6 + 5!$
$72 + 120 = 192$

72 The number of subsets containing exactly 4 elements of the set {2,4,6,8,10,12,14,16,18} is equal to

(A) 126

(B) 63

(C) 189

(D) 58

(E) 94

Ans: (A)

${}^{9}C_{4} = 126$

73 If ${}^{11}P_{r} = 7920$ and ${}^{11}C_{r} = 330$, then the value of r is equal to

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

Ans: (C)

$$\frac{n_{p_r}}{n_{c_r}} = r! \quad = \frac{7920}{330} = 24$$

$$r! \quad = 24$$

74 In the binomial expansion of $(x - 2y^2)^9$, the coefficient of x^6y^6 is equal to

(A) -672

(B) 672

(C) 336

(D) -336

(E) -512

Ans: (A)

$$^{T}3 + 1 = 9c_3(-2)^3x(y^2)^3$$

$$\frac{9 \times 8 \times y}{1 \times 2 \times 3} \times -8$$

$$\Rightarrow \ -24 \times 21$$

75 Let $(3 + x)^{10} = a_0 + a_1(1 + x) + a_2(1 + x)^2 + \cdots a_{10}(1 + x)^{10}$, where $a_1, a_2, \cdots a_{10}$ are constants. Then the value of $a_0 + a_1 + a_2 + \cdots a_{10}$ is equal to

(A) 2^{20}

(B) 2^{10}

(C) 3^{10}

(D) 2^{11}

(E) 2^{15}

Ans: (C)

76 If ${}^nC_5 + {}^nC_6 = {}^{51}C_6$, then the value of n is equal to

(A) 49

(B) 50

(C) 45

(D) 46

(E) 51

Ans: (B)

$^{n+1}C_6 = {}^{51}C_6$
$n = 50.$

77 Let $A = \begin{bmatrix} 3 & 4 \\ 1 & -2 \end{bmatrix}$ and let $AB = \begin{bmatrix} -5 & 41 \\ 5 & -13 \end{bmatrix}$. Then $|B^T| =$

(A) $\frac{1}{14}$

(B) 14

(C) 10

(D) -10

(E) -14

Ans: (B)

$|AB| = |A||B|$

$$|A| = -6 - 4 = -10$$
$$= 65 - 205$$
$$= -140$$

78 Let $A = \begin{vmatrix} 2 & 1 & -2 \\ 1 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix}$ and let $B = |A|\text{adj}\,(A)$. Then $|B| =$

(A) 256

(B) 64

(C) 512

(D) 1024

(E) 128

Ans: (D)

$$\begin{vmatrix} 0 & 1 & -2 \\ 0 & 1 & -1 \\ 4 & 0 & 3 \end{vmatrix} = 4(1) = 4$$

79 The values of x satisfying the equation $\begin{vmatrix} x & 4 & 0 \\ 2 & 2 & -x \\ 1 & 1 & 1 \end{vmatrix} = 0$ are (A) $2, -4$

(A) $2, -4$

(B) 1,2

(C) -1,2

(D) $-1, -2$

(E) -2,4

Ans: (E)

$$\begin{vmatrix} x-4 & 4 & 0 \\ 0 & 2 & -x \\ 0 & 1 & 1 \end{vmatrix}$$
$$(x-4)(2+x) = 0$$

80 If $A = [2 \quad 0 \quad 6]$ and $B = \begin{bmatrix} 3 & 5 \\ 7 & -2 \\ 6 & 6 \end{bmatrix}$, then $AB =$

(A) $[42 \quad 46]$

(B) $\begin{bmatrix} 42 \\ 46 \end{bmatrix}$

(C) $\begin{bmatrix} 6 & 10 \\ 0 & 0 \\ 36 & 36 \end{bmatrix}$

(D) $[17 \quad 19]$

(E) $\begin{bmatrix} 2 & 12 \\ 14 & -4 \end{bmatrix}$

Ans: (A)

$6 + 0 + 36 = 42.$

$[42 \; 46]$

81 If A is non-singular matrix and if $A^{-1} = \frac{1}{2}\begin{bmatrix} -10 & -4 \\ 2 & 1 \end{bmatrix}$, then adj $(A) =$

(A) $\begin{bmatrix} -1 & -4 \\ 2 & 10 \end{bmatrix}$

(B) $\begin{bmatrix} 10 & 4 \\ -2 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 4 \\ -2 & -10 \end{bmatrix}$

(D) $\begin{bmatrix} -10 & -4 \\ 2 & 1 \end{bmatrix}$

(E) $\begin{bmatrix} -1 & -4 \\ 10 & 2 \end{bmatrix}$

Ans: (B)

$\begin{bmatrix} 10 & 4 \\ -2 & -1 \end{bmatrix}$

82 $\begin{vmatrix} \sin\alpha & \cos(\alpha+\theta) & \cos\alpha \\ \sin\beta & \cos(\beta+\theta) & \cos\beta \\ \sin\gamma & \cos(\gamma+\theta) & \cos\gamma \end{vmatrix} =$

(A) -1

(B) 1

(C) 2

(D) 4

(E) 0

Ans: (E)

$$\frac{1}{\sin\theta\cos\theta}\begin{vmatrix} \sin\alpha & \sin\theta & \cos(\alpha+\theta) & \cos\alpha \\ \sin\beta & \sin\theta & \cos(\beta+\theta) & \cos\beta \\ \sin\gamma & \sin\theta & \cos(\gamma+\theta) & \cos\gamma \end{vmatrix}$$

83 The solution set of the inequality $-2 \leq \frac{3x+2}{2} < 7$ is

(A) $\{x: 3 \leq x < 4\}$

(B) $\{x: -2 \leq x < 3\}$

(C) $\{x: -2 \leq x < 4\}$

(D) $\{x: 0 \leq x < 6\}$

(E) $\{x: -2 \leq x < 6\}$

Ans: (C)

$-4 \leq 3x + 2 < 14$

$-6 \leqslant 3x < 12$

$-2 \leq x < 4$

84 The set of all x satisfying the inequality $|3x + 4| \leq 7$ is

(A) $\left[-1, \frac{11}{3}\right]$

(B) $\left[\frac{4}{3}, \frac{7}{3}\right]$

(C) $\left[\frac{-11}{3}, 1\right]$

(D) $\left[\frac{-4}{3}, \frac{7}{3}\right]$

(E) $\left[\frac{-4}{3}, \frac{11}{3}\right]$

Ans: (C)

$-7 \leq 3x + 4 \leq 7$

$-11 \leq 3x \leq 3$

$\frac{-11}{3} \leq x \leq 1$

$\left[\frac{-11}{3}, 1\right]$

85 If the solution set of the inequality $|a + 3x| \leq 6$ is $\left[\frac{-8}{3}, \frac{4}{3}\right]$, then the value of a is equal to

(A) -1

(B) -2

(C) 4

(D) -4

(E) 2

Ans: (E)

$-6 \leq a + 3x \leq 6$

$-a - 6 \leq 3x \leq 6 - a.$

$\frac{-a-6}{3} \leq x \leq \frac{6-a}{3}$

$a = 2.$

86 Consider the following statements :

(i) For every positive real number $x, x-10$ is positive.

(ii) Let n be a natural number. If n^2 is even, then n is even. f

(iii) If a natural number is odd, then its square is also odd.

Then

(A) (i) False, (ii) True and (iii) True

(B) (i) False, (ii) False and (iii) True

(C) (i) True, (ii) False and (iii) True

(D) (i) True, (ii) True and (iii) True

(E) (i) False, (ii) True and (iii) False

Ans: (A)

87 If $\cos\ \theta=\frac{5}{11}$ and $\tan\ \theta<0$, then the value of $\sin\ \theta$ is equal to

(A) $\frac{8\sqrt{6}}{11}$

(B) $\frac{-8\sqrt{6}}{11}$

(C) $\frac{4\sqrt{6}}{11}$

(D) $\frac{-4\sqrt{6}}{11}$

(E) $\frac{6}{11}$

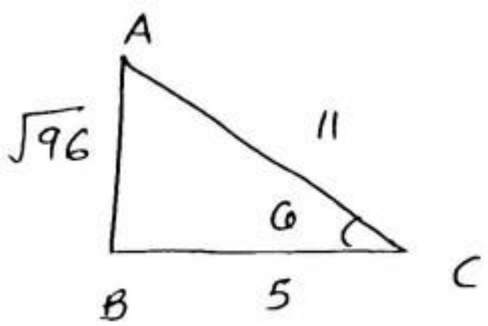
Ans: (D)

$$\text{Sin }\theta = \frac{-\sqrt{96}}{11}$$

$$AB = \sqrt{121-25}$$

$$= \sqrt{96}$$

$$\text{Sin } 6 = \frac{-4\sqrt{6}}{11}$$



88 If α and β are two acute angles of a right triangle, then $(\sin\ \alpha+\sin\ \beta)^2+(\cos\ \alpha+\cos\ \beta)^2=$

(A) $1+\sin\ 2\alpha$

(B) $2(1+\sin\ 2\alpha)$

(C) $1+\cos\ 2\alpha$

(D) $2(1+2\cos\ 2\alpha)$

(E) $2+\sin\ 2\alpha$

Ans: (B)

$2+2\cos(\alpha-\beta)$

$2(1+\cos(\alpha-\beta))$ $\alpha+\beta=90^\circ$

β $=90-\alpha.$

89 The range of the function $f(x)=2\sin(3x)+1$ is equal to

(A) $[-1,1]$

(B) $\left[\frac{-1}{3},\frac{1}{3}\right]$

(C) $[-2,1]$

(D) $[-1,2]$

(E) $[-1,3]$

Ans: (E)

$-2+1=-1$

$2+1=3$

$[-1,3]$

90 The period of the function $g(x)=5\cot\left(\frac{\pi}{3}x+\frac{\pi}{6}\right)+2$ is equal to

(A) 2

(B) 3

(C) 4

(D) 5

(E) 6

Ans: (B)

91 If $\theta\in(-\pi,0)$ and $\cos\ \theta=\frac{-12}{13}$, then $\sin\left(\frac{\theta}{2}\right)=$

(A) $\frac{-5\sqrt{26}}{26}$

(B) $\frac{5\sqrt{26}}{26}$

(C) $\frac{-5\sqrt{13}}{13}$

(D) $\frac{5\sqrt{13}}{13}$

(E) $\frac{-5\sqrt{13}}{26}$

Ans: (A)

$$\sin^2 \theta/2 = \frac{1-\cos\theta}{2} = \frac{1-12/13}{2}$$
$$= 25/13/2$$
$$= 25/26$$
$$\sin\varphi = \sqrt{25/26} = \pm 5/\sqrt{26}$$

92 The solutions of the equation $\cos\theta = 2 - 3\sin\left(\frac{\theta}{2}\right)$ in the interval $0 \leq \theta \leq \pi$ are

(A) $\frac{\pi}{4}, \pi$

(B) $\frac{\pi}{3}, \frac{\pi}{2}$

(C) $\frac{\pi}{3}, \pi$

(D) $\frac{\pi}{6}, \frac{\pi}{2}$

(E) $\frac{\pi}{6}, \pi$

Ans: (E)

$$1 - 2\sin^2\theta/2 = 2 - 3\sin\theta/2$$
$$1 - 2x^2 = 2 - 3x$$
$$x = \sin\theta/2$$
$$2x^2 - 3x + 1 = 0$$
$$2x^2 - 2x - x + 1 = 0$$
$$2x(x-1) - (x-1) = 0$$

93 The value of $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$ is equal to

(A) $\frac{7\pi}{6}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

(E) $\frac{5\pi}{6}$

Ans: (E)

$\pi + \pi/6$

$$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

94 The value of $\tan\left(\sin^{-1}\left(\frac{7}{25}\right)\right)$ is equal to

(A) $\frac{18}{25}$

(B) $\frac{24}{25}$

(C) $\frac{7}{24}$

(D) $\frac{3}{4}$

(E) $\frac{7}{18}$

Ans: (C)

$\sin^{-1}(7/25) = \theta.$
$\sin\ \theta = 7/25$
$\tan\ \theta = 7/24$
$\tan^{-1}(\tan(7/24))$

95 $\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{200}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{200}\right)\right) =$

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$

(D) 1

(E) 0

Ans: (E)

$\sin^{-1} x + \cos^{-1} x = \pi/2.$

96 The equation of the straight line parallel to $y = -3x$ and passing through the point $(3, -2)$ is

(A) $y = -3x + 7$

(B) $y = -3x + 9$

(C) $y = -3x - 11$

(D) $y = -3x - 7$

(E) $y = -3x + 11$

Ans: (A)

$y = mx + c$
$y = -3x + c$
$(3,2)\ 2 = -3 \times 3 + c$
$2 = -9 + c$

c =7

97 The intercepts of a line with coordinate axes are equal. If the line passes through (2,3), then its equation is

(A) $2x + 3y = 5$

(B) $x + y = 5$

(C) $5x + 5y = 1$

(D) $x + y = 6$

(E) $3x + 2y = 5$

Ans: (B)

$\frac{x}{a} + \frac{y}{a} = 1$

$x + y = a.$

$(2,3) \rightarrow 2 + 3 = a$

$a = 5.$

98 If the line $y = mx + c$ is perpendicular to $y = 1 + x$ and passes through the point (1,2), then the value of c is equal to

(A) 1

(B) -1

(C) -3

(D) 3

(E) 0

Ans: (D)

$\mathrm{y} = mx + c$

$\mathrm{y} = x + 1$

slope $= 1$

$\mathrm{y} = mx + c = -x + c$

Perpendicular line $m = -1$

$2 = -1 + c$

$\mathrm{c} = 2 + 1$

$= 3$

99 Let $A(-1,2), B(1,3)$ and $C(a, b)$ be collinear. If B divides AC such that $BC = 8AB$, then the coordinates of C are

(A) $\left(\frac{5}{4}, \frac{25}{8}\right)$

(B) (17,9)

(C) (17,11)

(D) $\left(\frac{5}{4}, \frac{5}{8}\right)$

(E) (1,11)

Ans: (C)

$$\frac{mx_2 + nx_1}{m+n} = 1$$

$$\frac{a \pm 8}{1+8} = 1$$

$$a - 8 = 9$$

$$a = 17$$

Similarly b =11

100 If the lines $2x - 3y + 5 = 0, 9x - 5y + 14 = 0$ and $3x - 7y + \lambda = 0$ are concurrent, then the value of λ is equal to

(A) 7

(B) 8

(C) 10

(D) 9

(E) 6

Ans: (C)

$$\begin{vmatrix} 2 & -3 & 5 \\ 9 & -5 & 14 \\ 3 & -7 & \lambda \end{vmatrix} = 0$$

$$\lambda = 10$$

101 The points of intersection of the line $y = x + 2$ and the circle $(x-2)^2 + y^2 = 16$ are

(A) $(-2,0), (2,4)$

(B) $(-2,4), (2,0)$

(C) $(4,0), (4,2)$

(D) $(4,6), (4,-2)$

(E) $(4,0), (4,-2)$

Ans: (A)

$$(x-2)^2 + (x+2)^2 = 16$$

$$2\{x^2 + 4\} = 16$$

$$x^2 + 4 = 8$$

$$x = 2, -2$$

102 The three vertices of a triangle are (0,0), (3,1) and (1,3). If this triangle is inscribed in a circle, then the equation of the circle is

(A) $2x^2 + 2y^2 - 2x - 6y = 0$

(B) $x^2 + y^2 - 3x - y = 0$

(C) $x^2 + y^2 - x - 3y = 0$

(D) $2x^2 + 2y^2 - 6x - 2y = 0$

(E) $2x^2 + 2y^2 - 5x - 5y = 0$

Ans: (E)

$x^2 + y^2 + 2gx + 2fy = 0$

103 The equation of the circle touching the x-axis at $(5,0)$ and the line $y = 10$ is

(A) $x^2 + y^2 - 10x - 10y + 25 = 0$

(B) $x^2 + y^2 - 10x - 10y - 25 = 0$

(C) $x^2 + y^2 - 5x - 5y - 5 = 0$

(D) $x^2 + y^2 - 5x - 5y + 5 = 0$

(E) $x^2 + y^2 + 10x + 10y - 25 = 0$

Ans: (A)

$$(x-5)^2 + (y-5)^2 = 5^2$$
$$x^2 - 10x + 25 + y^2 - 10y + 25 = 25$$
$$x^2 + y^2 - 10x - 10y + 25 = 0$$

104 If the radius of the circle $x^2 + y^2 + ax + by + 3 = 0$ is 2 , then the point (a, b) lies on the circle

(A) $x^2 + y^2 = 7$

(B) $x^2 + y^2 = 4$

(C) $x^2 + y^2 = 14$

(D) $x^2 + y^2 = 28$

(E) $x^2 + y^2 = 1$

Ans: (D)

$$\sqrt{g^2 + f^2 - c} = \gamma$$
$$\sqrt{\left(\frac{-a}{2}\right)^2 + \left(\frac{-b}{2}\right)^2 - 3} = 2$$
$$\frac{a^2}{4} + \frac{b^2}{4} - 3 = 4$$
$$a^2 + b^2 = 28$$

105 If the line $2x - 3y + c = 0$ passes through the focus of the parabola $x^2 = -8y$, then the value of c is equal to

(A) 4

(B) -6

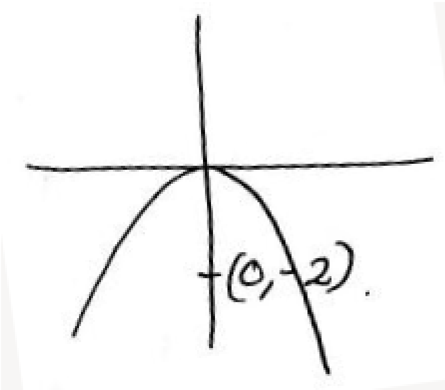
(C) 6

(D) -4

(E) 2

Ans: (B)

$x^2 = -4ay$
$a = 2$
$0 + 6 + c = 0$
$c = -6.$



106 The centre of the ellipse $x^2 + 7y^2 - 14x + 28y + 49 = 0$ is

(A) (7,0)

(B) $(7, -4)$

(C) $(7, -2)$

(D) $(-7,4)$

(E) $(-7,2)$

Ans: (C)

$x^2 - 14x + 7y^2 + 28y + 49 = 0$
$(x-7)^2 + 7((y+2)^2 - 4) + 49 = 0.$
$(x-7)^2 + 7(y+2)^2 - 28 + 49 = 0.$

107 The end points of the major axis of an ellipse are (2,4) and $(2, -8)$. If the distance between foci of this ellipse is 4 , then the equation of the ellipse is

(A) $\frac{(x-2)^2}{32} + \frac{(y+2)^2}{36} = 1$

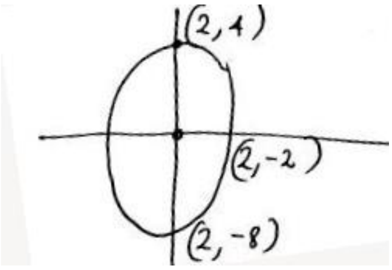
(B) $\frac{(x-4)^2}{32} + \frac{(y+2)^2}{36} = 1$

(C) $\dfrac{(x-2)^2}{36}+\dfrac{(y+2)^2}{32}=1$

(D) $\dfrac{(x-2)^2}{32}+\dfrac{(y-4)^2}{36}=1$

(E) $\dfrac{(x-2)^2}{36}+\dfrac{(y-4)^2}{32}=1$

Ans: (A)



$$\begin{aligned} 2a &= 12 \\ a &= 6 \\ 2c &= 4 \\ c &= 2 \\ b^2 &= 36-4 \\ &= 32 \end{aligned}$$

$$\frac{(y+2)^2}{a^2}+\frac{(x-2)^2}{b^2}=1$$

108 If $(-1,0)$ and $(3,0)$ are foci of an ellipse and the length of the major axis is 6 , then the length of the minor axis is

(A) $\sqrt{5}$

(B) 5

(C) 10

(D) $2\sqrt{5}$

(E) 3

Ans: (D)

$$\begin{aligned} 2c &= 4 \quad \Rightarrow c = 2 \\ 2a &= 6 \quad \Rightarrow a = 3 \\ b^2 &= a^2-c^2 = 9-4 = 5 \end{aligned}$$

$$\begin{aligned} b &= \sqrt{5} \\ 2b &= 2\sqrt{5} \end{aligned}$$

109 The eccentricity of the hyperbola $\frac{(x-3)^2}{9}-\frac{4(y-1)^2}{45}=1$ is equal to

(A) $\frac{3}{\sqrt{5}}$

(B) $\frac{5}{3}$

(C) $\frac{5}{\sqrt{3}}$

(D) $\frac{5}{2}$

(E) $\frac{3}{2}$

Ans: (E)

$$\frac{(x-3)^2}{9} - \frac{(y-1)^2}{45/4} = 1$$

$$e = c/a = \frac{\sqrt{a^2+b^2}}{a}$$

$$= \sqrt{\frac{9+45/4}{3}}$$

$$= \sqrt{\frac{\frac{36+45}{4}}{3}}$$

110 If $\vec{a} \times \vec{b} = 7\hat{\imath} + 9\hat{\jmath} + 10\hat{k}$ and $\vec{a} \cdot \vec{b} = -20$, then $|\vec{a}|^2|\vec{b}|^2 =$

(A) 530

(B) 580

(C) 400

(D) 630

(E) 560

Ans: (D)

$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 \sin^2 Q.$

$$\frac{|\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 \cos^2 a.}{49 + \vec{b}\big|^2 + \big|\vec{a} \cdot \vec{b}\ \ \big|^2 = |\vec{a}|^2\big|\vec{b}\ \ \big|^2}$$

$$49 + 81 + 100 + 400 = |\vec{a}|^2\big|\vec{b}\ \big|^2$$
$$= 630$$

111 Let $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{a} + \vec{b} = 4\hat{\imath} - 2\hat{\jmath} + \lambda\hat{k}$. If $\vec{a} \cdot \vec{b} = 4$, then the value of λ is equal to

(A) 3

(B) -3

(C) -6

(D) 6

(E) 0

Ans: (C)

$\vec{a} = i + 2j - 3k$
$\vec{b} = 3i - 4j + (\lambda + 3)k$
$\vec{a} \cdot \vec{b} = 4 \Rightarrow 3 - 8 - 3(\lambda + 3) = 4$
$\lambda = -6.$

112 If $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{10}, |\vec{a} - \vec{b}| = \sqrt{24}$ and θ is angle between $\vec{a}$ and $\vec{b}$, then $\cos\ \theta =$

(A) $\frac{\sqrt{35}}{70}$

(B) $\frac{\sqrt{6}}{12}$

(C) $\frac{\sqrt{15}}{60}$

(D) $\frac{\sqrt{210}}{35}$

(E) 0

Ans: (E)

$14 + 10 - 2\vec{a} \cdot \vec{b} = 24$
$\vec{a} \cdot \vec{b} = 0.$

113 If $|\vec{a}| = 10$ and $|\vec{b}| = 5$, then the value of $(\vec{a} + 2\vec{b}) \cdot (\vec{a} - 2\vec{b})$ is equal to

(A) 32

(B) 16

(C) 8

(D) 4

(E) 0

Ans: (E)

$|\vec{a}|2 - 4\left|\vec{b}'\right|^2$
$100 - 4 \times 25 = 0$

114 If $\vec{a} = \hat{\imath} - 3\hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$, then the value of $(\vec{a} \times \vec{b}) \cdot \vec{b}$ is equal to

(A) 3

(B) -3

(C) 7

(D) -7

(E) 0

Ans: (E)

0

115 If $\vec{a}$ and $\vec{b}$ are position vectors of the points $(\alpha, 3,0)$ and $(1,0,0)$ respectively and if the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$, then the value of α is equal to

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

Ans: (C)

$$\vec{a} = \alpha i + 3j + 0k$$
$$\vec{b} = i + 0j + 0k$$
$$\frac{1}{\sqrt{2}} = \frac{\alpha}{\sqrt{\alpha^2 + 9 + 0} \cdot \sqrt{1}}$$
$$\alpha^2 + 9 = 2\alpha^2$$
$$\alpha^2 = 9$$
$$\alpha = \pm 3.$$

116 If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$ and $\vec{b} = \hat{\imath} + 3\hat{\jmath} + 2\hat{k}$, then a unit vector in the direction of $\vec{a} + \vec{b}$ is

(A) $\frac{1}{6}(3\hat{\imath} + 6\hat{\jmath} - 2\hat{k})$

(B) $\frac{1}{\sqrt{70}}(3\hat{\imath} + 6\hat{\jmath} - 5\hat{k})$

(C) $\frac{1}{7}(3\hat{\imath} + 6\hat{\jmath} - 2\hat{k})$

(D) $\frac{1}{\sqrt{50}}(3\hat{\imath} + 6\hat{\jmath} - 3\hat{k})$

(E) $\frac{1}{\sqrt{6}}(\hat{\imath} + 2\hat{\jmath} - \hat{k})$

Ans: (C)

$$\sqrt{9 + 36 + 4} = 7$$
$$\frac{1}{7}(3i + 6j - 2k)$$

117 If $|\vec{u}| = 3, |\vec{v}| = 2$ and $|\vec{u} \times \vec{v}| = 3$, then the angle between $\vec{u}$ and $\vec{v}$ is equal to

(A) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$

(B) $\frac{\pi}{6}$ or $\frac{5\pi}{6}$

(C) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$

(D) $\frac{\pi}{2}$

(E) 0

Ans: (B)

$$|\vec{u}||\vec{v}|\sin\,\theta = 3$$

$$\sin\,\theta = \frac{3}{3 \times 2} = \frac{1}{2}\ \theta = \pi/6$$

118 The equation of the plane passing through the point $(-1, -2, -3)$ and perpendicular to the x-axis is

(A) $x = -1$

(B) $y = -2$

(C) $z = -3$

(D) $2x + 3y = 5$

(E) $x + y + z = 6$

Ans: (A)

$$1(x + 1) = 0$$

$$x = -1$$

119 Let L_1 be the line joining (0,0,0) and (1,2,3) and L_2 be the line joining (2,3,4) and (3,4,5). The point of intersection of L_1 and L_2 is

(A) (0,0,0)

(B) (1,2,3)

(C) (2,3,4)

(D) (3,4,5)

(E) (1,1,1)

Ans: (B)

$$L_1: \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$$

$$L_2: \frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{1}$$

120 The equation of the line through the point $(1, -1,1)$ and parallel to the line joining the points $(-2,2,0)$ and $(-1,1,1)$ is

(A) $\frac{x-1}{-3} = \frac{y-1}{-1} = z-1$

(B) $1-x = 1+y = 1-z$

(C) $x+1 = -(y-1) = z-1$

(D) $\frac{x-1}{-1} = \frac{y+1}{2} = \frac{z-1}{1}$

(E) $x+2 = y-2 = z$

Ans: (B)

$$\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-1}{1}$$
$$1-x = -(y+1) = 1-z$$