B1

XYLEM

KEAM 2023 - PAPER II

1. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x)=x^2+9$. The range of f is

(A) $\mathbb{R}$

(B) $(-\infty,-9]\cup[9,\infty)$

(C) $[9,\infty)$ (D) $[3,\infty)$

(E) $[3,\infty)\cup(-\infty,-3]$

Ans: C

$$f(x)=x^2+9$$

minimum value of $f(x)=9$

$$\therefore \quad \text{Range } =[9,\infty)$$

2. Let $f(x)=\dfrac{x-1}{x+1}$. Let $S=\{x\in\mathbb{R}\mid fof^{-1}(x)=x$ does not hold $\}$. The cardinality of S is

(A) a finite number, but not equal to 1, 2, 3

(B) 3

(C) 2

(D) 1

(E) infinite

Ans: D

Domain of $f=A=\mathbb{R}-\{-1\}$

Range of $f=B=\mathbb{R}-\{1\}$

$fof^{-1}=I_B$

So $\quad I_B(x)=x \quad \forall x\in B$

$\Rightarrow \quad S=B'=\{1\}$

$n(S)=1$

3. The domain of the real valued function $f(x)=\sqrt{x^2-4}+\dfrac{1}{\sqrt{x^2-7x+6}}$ is

(A) $\mathbb{R}-[-6,-2)$ (B) $\mathbb{R}-[-6,2)$

(C) $\mathbb{R}-[-2,6)$ (D) $\mathbb{R}-(2,6]$

(E) $\mathbb{R}-(-2,6]$

Ans: E

$f(x)=\sqrt{x^2-4}+\dfrac{1}{\sqrt{x^2-7x+6}}$

$x^2-4\geq 0-(1)$

$x^2-7x+6>0-(2)$

$x\in(-\infty,-2]\cup[2,\infty)$ and

$x\in(-\infty,1)\cup(6,\infty)$

$\Rightarrow x\in(-\infty,-2]\cup(6,\infty)$

$R-(-2,6]$

4. The number of solutions of the equation $\frac{1}{2}\left(x^3+1\right)=\sqrt[3]{2x-1}$ is

(A) 0 (B) 6

(C) 9 (D) Infinite

(E) 3

Ans: E

$\frac{1}{2}\left(x^3+1\right)=(2x-1)^{1/3}$

$f(x)=\frac{1}{2}\left(x^3+1\right)$

$f^{-1}(x)=(2x-1)^{1/3}$

$f(x)=f^{-1}(x)$ will intersect where

$y=x$

$\frac{1}{2}\left(x^3+1\right)=x$

$\Rightarrow x^3+1=2x$

$x^3-2x^3+1=0$

$(x-1)\left(x^2+x-1\right)=0$

$\Rightarrow x = 1, x = \frac{-1}{2} \pm \frac{\sqrt{5}}{2}$
3 points

5. Let a, b, c, d be an increasing sequence of real numbers, which are in geometric progression. If $a + d = 112$ and $b + c = 48$, then the value of $\frac{a+c+8}{b}$ is

   (A) 1 (B) 5
   (C) 4 (D) 3
   (E) 2

   Ans: C
   $a + d = 112$

   $b + c = 48$

   $\Rightarrow \quad a + ar^3 = 112$

   $a\left(1 + r^3\right) = 112$

   $ar + ar^2 = 48$

   $a\left(r + r^2\right) = 48$

   $$\frac{1 + r^3}{r + r^2} = \frac{112}{48} = \frac{7}{3}$$
   $$\frac{1 + r + r^2}{r} = \frac{7}{3}$$

   $3 + 3r + 3\theta^2 = 7r$

   $300^2 - 4r + 3 = 0$

   $r = 3, r = 1/3$

   $r = 3 \Rightarrow a\left(3 + 3^2\right) = 48$

   $12a = 48$

   $a = 4$

   sequence becomes $4, 12, 36, 108$

   $$\therefore \frac{a + c + d}{b} = \frac{4 + 36 + 8}{12} = 4$$

6. Let a, b be two real numbers between 3 and 81 such that the resulting sequence $3, a, b, 81$ is in a geometric progression. The value of $a + b$ is

   (A) 29 (B) 90
   (C) 27 (D) 81
   (E) 36

   $a = r, b = 27$

   Ans: E $\quad a + b = 36$

   (E)

7. Let $a_1, a_2, a_3, \ldots$ be an increasing sequence of natural numbers, which are in an arithmetic progression with common difference d. Suppose $a_1 + a_2 + a_3 = 27$ and $a_1^2 + a_2^2 + a_3^2 = 275$. Then the values of a_1, d are

   (A) $a_1 = 3; d = 2$ (B) $a_1 = -5; d = 4$
   (C) $a_1 = 4; d = 5$ (D) $a_1 = -4; d = 5$
   (E) $a_1 = 5; d = 4$

   Ans: E
   $a_1 + a_2 + a_3 = 27$

   $a_1^2 + a_2^2 + a_3^2 = 275$

   $a_1 = a_2 - d$

   $a_3 = a_2 + d,$
   $3a_2 = 27 \Rightarrow a_2 = 9$
   $(a_2 - d)^2 + a_2^2 + (a_2 + d)^2$

   $(a - d)^2 + a^2 + (a + d)^2 = 275$

   $2\left(a^2 + d^2\right) = 275 - 81$

   $a^2 + d^2 = 97$

   $d^2 = 16$

   $d = 4$

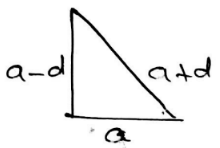
   $a = 5, d = 4$

   (E)

8. The sides of a right-angled triangle are in an arithmetic progression. If the area of the triangle is 54 , then the length of the longest side is

   (A) 6 (B) 12
   (C) 15 (D) 9
   (E) 18

   Ans: C



$(a-d)^2 + a^2 = (a+d)^2$

$a^2 - 2ad + d^2 + a^2 = (a+d)^2$

$= a^2 + 2ad + d^2$

$a^2 = 4ad$

$a = 4d$

$4d(3d) = 108$

$d^2 = \frac{108}{12} = 9$

$d = 3$

$a = 12$

lengths of sides will be $9, 12, 15$ (c)

9. Let A be $(2n+1) \times (2n+1)$ matrix with integer entries and positive determinant, where $n \in \mathbb{N}$. If $AA^T = I = A^T A$, then which of the following statements always holds?

   (A) $\det(A) = 0$

   (B) $\det(A+I) \neq 0$

   (C) $\det(A+I) = 0$

   (D) $\det(A-I) = 0$

   (E) $\det(A-I) \neq 0$

   Ans: D

   $|A-I| = \left|A - AA^\top\right|$

   $= |A| \left|I - A^\top\right|$

   $= |A||I-A| \ldots\ldots (1)$

   Since $\left(AA^T\right| = 1$ and $|A| = +ve$

$$|A| = 1$$

Substitute in (1),

$$|A-I| = |I-A|$$
$$\Rightarrow |A-I| = |-(I-A)|$$
$$= (-1)^{2n+1}|A-I|$$
$$= -|A-I|$$
$$\Rightarrow |A-I| = 0$$

10. The inequality $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{(2-x)}{5}$ holds for x belonging to

    (A) $\mathbf{R}$ (B) $(-\infty, 3]$

    (C) $(-\infty, -3] \cup [3, \infty)$

    (D) $(-\infty, 2]$

    (E) $(-\infty, 2] \cup [4, \infty)$

    Ans: D

$$\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{(2-x)}{5}$$

$20(2x-1) \geq 15(3x-2) - 12(2-x)$

$$40x - 20 \geq 45x - 30 - 24 + 12x$$
$$34 \geq 1 > x$$
$$x \leq 2$$
$$x \in (-\infty, 2]$$

11. The contrapositive of the statement "If the number is not divisible by 3 , then it is not divisible by 15$''$ is

    (A) If the number is not divisible by 3 , then it is not divisible by 15

    (B) If the number is not divisible by 15 , then it is not divisible by 3

    (C) If the number is not divisible by 15 , then it is divisible by 3

    (D) If the number is divisible by 15 , then it is divisible by 3

    (E) If the number is divisible by 15 , then it is not divisible by 3

    Ans: D

    controapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim q$

12. Let A be an invertible matrix of size 4×4

with complex entries. If the determinant of $adj(A)$ is 5 , then the number of possible value of determinant of A is

(A) 1 (B) 4

(C) 6 (D) 3

(E) 2

Ans: D

$$|\operatorname{adj} A| = 5$$
$$|\operatorname{adj} A| = |A|^{n-1} = |A|^3$$
$$|A|^3 = 5$$

$\Rightarrow |A|$ can have 3 values since it is a cubic equation.

13. The determinant of the matrix $\begin{bmatrix} 1 & 4 & 8 \\ 1 & 9 & 27 \\ 1 & 16 & 64 \end{bmatrix}$ is

(A) 13 (B) 208

(C) 104 (D) 26

(E) 52

Ans: E

$$\begin{vmatrix} 1 & 4 & 8 \\ 1 & 9 & 27 \\ 1 & 16 & 64 \end{vmatrix}$$
$$= (64 \times 9 - 27 \times 16) - 4(64 \times -27)$$
$$+ 8(16 - 9)$$
$$= (576 - 432) - 4 \times 37 + 56$$
$$= 144 - 148 + 56$$
$$= 52$$

14. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \cdot adj A = AA^T$, then which of the following statements is true

(A) $5a - b = -5$ (B) $5a + b = 10$

(C) $\det(A) < 0$ (D) A is symmetric

(E) $\det(A) \geq 0$

Ans: E

$$A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$$
$$adj A = \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix}$$
$$A^\top = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$$
$$A \cdot adj A = \begin{bmatrix} 10a + 3b & 0 \\ 0 & 10a + 3b \end{bmatrix}$$
$$AA^\top = \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$$

Since $A \cdot \text{ad}; A = AA^\top$

$15a - 2b = 0$ ......(1)& $10a + 3b = 13$....(2)

on solving a (1) and (2)

$a = \frac{2}{5}$

$b = 3$

$$\therefore \quad A = \begin{bmatrix} 5 \times \frac{2}{5} & -3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$$

$|A| = 4 - -9 = 13 \geq 0$

$\therefore \det A \geq 0$

15. Suppose $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is an adjoint of the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$. The value of $\frac{a_1 + b_2 + c_3}{b_1 a_2}$ is

(A) 0 (B) 3

(C) 1 (D) 2

(E) 4

Ans: B

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = adj \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\therefore \frac{a_1 + b_2 + c_3}{b_1 a_2} = \frac{7+1+1}{-3 \times -1} = \frac{9}{3} = 3$$

16. If $x + iy = \frac{1}{(1+\cos\theta)+i\sin\theta}$, then the value of x^2+1 is

(A) $\frac{7}{4}$ (B) $\frac{13}{4}$

(C) $\frac{1}{4}$ (D) $\frac{9}{4}$

(E) $\frac{5}{4}$

Ans: E

(e) $x + iy$

$= \frac{1}{(1+\cos\theta)+i\sin\theta} \times \frac{(1+\cos\theta)-i\sin\theta}{(1+\cos\theta)-i\sin\theta}$

$\Rightarrow x = \frac{1}{2}$

$x^2 + 1 = \frac{1}{4} + 1 = \frac{5}{4}$

17. If α, β, γ are the cube roots of -2 , then the value of $\frac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}$ is $(x, y, z$ are variables)

(A) $e^{i\pi/3}$ (B) $e^{2\pi i/3}$

(C) 1 (D) -1

(E) $e^{4\pi i/3}$

Ans: E

$$\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha}$$

$$= \frac{x(-2)^{1/3} + y\left(-2^{1/3}\omega\right) + z\left(-2^{1/3}\omega^2\right)}{x\left(-2^{1/3}\omega\right) + y\left(-2^{1/3}\omega^2\right) + z(-2)^{1/3}}$$

$$= \frac{-2^{1/3}\left(x + y\omega + z\omega^2\right)\omega}{-2^{1/3}\left(x\omega + y\omega^2 + z\right)\omega}$$

$$= \frac{1}{\omega} = \frac{\omega^3}{\omega} = \omega^2$$

$$= e^{\frac{4\pi i}{3}}$$

18. Let $x + \frac{1}{x} = 2\cos\alpha$. For any $n \in \mathbb{N}$, the value of $x^n - \frac{1}{x^n}$ is

(A) $\cos(n\alpha)$ (B) $2\cos(n\alpha)$

(C) $2i\sin(n\alpha)$ (D) $i\sin(n\alpha)$

(E) $4\cos(n\alpha)$

Ans:C

$x + \frac{1}{x} = 2\cos\alpha$

$x = \cos\alpha + i\sin\alpha$

$x^n = \cos(n\alpha) + i\sin(n\alpha)$

$\frac{1}{x^n} = \cos(n\alpha) - i\sin(n\alpha)$

$\therefore x^n - \frac{1}{x^n} = 2i\sin(n\alpha)$

19. If $f(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1 z^n + a_0 \in \mathbb{R}[z]$ is a polynomial in z with no root over $\mathbb{R}$, then $\deg(f)$ is

(A) 9

(B) always ≤ 4

(C) an odd number

(D) always ≥ 4

(E) an even number

Ans: E

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$$f(z) = a_n z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0$$

have only complex roots So they occur in congugate pairs. $\therefore$ degree should be even

20. Let $S = \{n \in \mathbb{N} \mid n^3 + 3n^2 + 5n + 3$ is not divisible by $3\}$. Then, which of the following statements is true about S

(A) $S = \phi$

(B) $|S| \geq 2$ and $|S|$ is a multiple of 5

(C) S is non-empty but $|S|$ is finite

(D) $|S|$ is infinite

(E) S is non-empty and $|S|$ is a multiple of 3

Ans:E

$S = \{n \in N : n^3 + 3n^2 + Sn + 3$ is not divisible by $3\}$

$n^3 + 3n^2 + 5n + 3$ is always divisible by 3

$$\therefore \quad s = \phi$$

21. If the coefficients of $(5r+4)^{th}$ term and $(r-1)^{th}$ term in the expansion of $(1+x)^{25}$ are equal, then r is

(A) 6 (B) 3
(C) 5 (D) 2
(E) 4

Ans: E

$$\begin{aligned}(5r+4)^{\text{th}} \text{ term } &= 25C_{5r+3}x^{5r+4}\\ (r-1)^{1/4} \text{ term } &= 25C_{r-2}x^{r-1}\end{aligned}$$

Given

$$25C_{5_r+3} \ \ x^{5r+4} = 25C_{r-2}$$
$$(5r+3)+(r-2) = 25$$
$$6r+1 = 25$$
$$6r = 24$$
$$r = 4$$

22. $\dfrac{\sum\limits_{r=0}^{n}(4r+3)\cdot({}^nc_r)^2}{(2n+3)}$ is For any $n \geq 0$, the value of

(A) ${}^{2n}C_{n-1}$ (B) ${}^{8n}C_n$
(C) ${}^{2n}C_{n+1}$ (D) ${}^{n}C_{n-2}$
(E) ${}^{2n}C_n$

Ans: E

$$\frac{\sum\limits_{r=0}^{n}(4r+3)\,({}^nC_r)^2}{2n+3}$$

take $n = 1$
$\therefore$ Given problem gives $\frac{3+7}{5} = 2$.
$\therefore$ we get ${}^{2n}C_n$

23. The number of ways in which we can distribute n identical balls in k boxes is

(A) ${}^nC_k$ (B) ${}^nC_{(k-1)}$
(C) ${}^{(n+k-1)}C_{(k-1)}$ (D) ${}^{(n-1)}C_{(k-1)}$
(E) ${}^{(n+k)}C_n$

Ans:C
Standard Result.

$${}^{(n+k-1)}C_{(k-1)}$$

24. Suppose there are 5 alike dogs, 6 alike monkeys and 7 alike horses. The number of ways of selecting one or more animals from these is

(A) 362 (B) 363
(C) 336 (D) 335
(E) 337

Ans: D
no dog, one dog. 2 dogs, $\cdots$5 dogs $\to$ 6 ways
no monkey, one monkey, 2 monkeys, $\cdots$6 monkeys $\to$ 7 ways
no horse, one horse, 2 horses,... 7 horses $\to$ 8 ways

$$\therefore 6 \times 7 \times 8 = 336$$

Number of ways of selecting one or more animals

$$= 336 - 1$$
$$= 335$$

[deleting the case no dog, no horse, no monkey]

25. Consider the following Linear Programming Problem (LPP) :

$$\text{Maximize } Z = 60x_1 + 50x_2$$
subject to
$$x_1 + 2x_2 \leq 40$$
$$3x_1 + 2x_2 \leq 60$$
$$x_1, x_2 \geq 0$$

Then, the

(A) LPP has a unique optimal solution.

(B) LPP is infeasible.

(C) LPP is unbounded.

(D) LPP has multiple optimal solutions.

(E) LPP has no solution.

Ans: A

Maximize $z = 60x_1 + 50x_2$

subject to $x_1 + 2x_2 \leq 40$

$3x_1 + 2x_2 \leq 60$

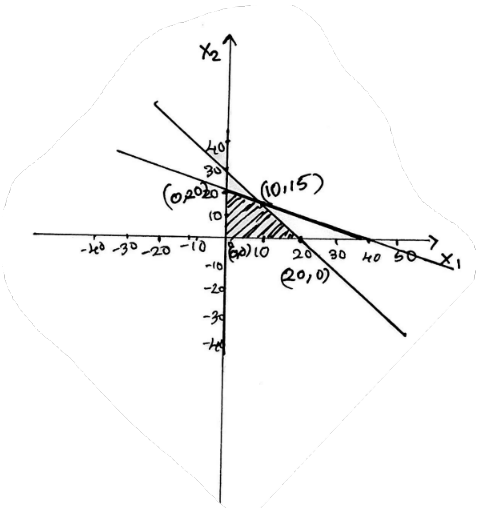
$x_1, x_2 \geq 0$

$3x_1 + 2x_2 = 60$

x_1	x_2	$P(x_1, x_2)$
0	30	$(0, 30)$
20	0	$(20, 0)$

$x_1 + 2x_2 = 40$

x_1	x_2	$P(x_1, x_2)$
0	20	$(0, 20)$
40	0	$(40, 0)$



$P(x_1, x_2)$	$Z = 60x_1 + 50x_2$
$(0, 20)$	$60 \times 0 + 50 \times 20 = 1000$
$(10, 15)$	$60 \times 10 + 50 \times 15 = 1350$
$(20, 0)$	$60 \times 20 + 50 \times 0 = 1200$
$(0, 0)$	$60 \times 0 + 50 \times 0 = 0$

therefore LPP has a unique optimal solution

26. Consider the linear programming problem :

Minimize $3x_1 + 4x_2 + 2x_3$
subject to

$$x_1 + x_2 + x_3 \leq 6$$
$$x_1 + 2x_2 + x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

Then, the number of basic solutions are

(A) 7 (B) 9

(C) 10 (D) 8

(E) 3

Ans:

27. In a linear programming problem, the restrictions under which the objective function is to be optimised are called as

(A) decision variables

(B) objective function

(C) constraints

(D) integer solutions

(E) optimal solutions

Ans: C

In a linear Programming problem, the restrictions under which the objective function is to be optimised are called Constraints.

28. Which of the following is the correct formulation of linear programming problem

(A) Max $Z = 2x_1 + x_2$; subject to $x_1 + x_2 \leq 10; x_1 \leq 3; x_1 \geq 0; x_2 \leq 0$

(B) Max $Z = 3x_1 + 2x_2$; subject to $x_1 + 2x_2 \geq 11; 3x_1 + x_2 \geq 24; x_1, x_2 \leq 0$

(C) Min $Z = x_1 + 5x_2$; subject to $2x_1 + 5x_2 \leq 10; x_1 + 3x_2 \leq 9; x_1, x_2 \geq 0$

(D) Min $Z = 4x_1 + 3x_2$; subject to $x_1 + 9x_2 \geq 8; 2x_1 + 5x_2 \leq 9; x_1 \leq 0, x_2 \geq 0$

(E) Max $Z = 2x_1 + 5x_2$; subject to $4x_1 + 9x_2 \leq 8; 2x_1 + 3x_2 \leq 9; x_1, x_2 \leq 0$

Ans: C

only option (C) satisfies the Non-negativity constraints.

$$\min z = x_1 + 5x_2$$

Subject to

$$2x_1 + 5x_2 \leq 10$$
$$x_1 + 3x_2 \leq 9$$
$$x_1, x_2 \geq 0$$

29. Let A and B be two independent events such that the odds in favour of A and B are 1:1 and 3 : 2, respectively. Then the probability that only one of the two occurs is

(A) 0.6 (B) 0.7

(C) 0.8 (D) 0.5

(E) 0.4

Ans: D

$P(A) = \dfrac{1}{2}$

$P(B) = \dfrac{3}{5}$

$$P(A \cap B' \text{ or } A' \cap B) = \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{3}{5} = \frac{1}{5} + \frac{3}{10} = \frac{5}{10} = \underline{\underline{0.5}}$$

30. A six faced fair die is rolled for a large number of times. Then, the mean value of the outcomes is

(A) 4.5 (B) 2.5

(C) 3.5 (D) 1.5

(E) 3

Ans: C

$x =$ outcome of dice $= \{1, 2, 3, 4, 5, 6\}$

(x)	1	2	3	4	5	6
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Mean, $E(x) = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$

$= \frac{1+2+3+4+5+6}{6}$

$= \frac{21}{6}$

$= 3.5$

31. Let the probability distribution of random variable X be

X	-2	-1	1	2	3
P(X $= x$)	k	$2k$	$2k$	k	$3k$

Then, the value of $E(X^2)$ is

(A) $\dfrac{19}{9}$ (B) $\dfrac{13}{3}$

(C) $\dfrac{35}{9}$ (D) $\dfrac{11}{3}$

(E) $\dfrac{7}{3}$

Ans: B

x	-2	-1	1	2	3
$P(X = x)$	k	$2k$	$2k$	k	$3k$

$\sum P(x) = 1$

$k + 2k + 2k + k + 3k = 1$

$9k = 1$

$k = \frac{1}{9}$

$E(x^2) = 4 \times k + 1 \times 2k + 1 \times 2k + 4 \times k + 9 \times 3k$

$= 4k + 2k + 2k + 4k + 27k$

$= 39k$

$= 39 \times \frac{1}{9} = \frac{13}{3}$

32. Let the standard deviation of x_1, x_2 and x_3 be 9 . Then, the variance of $3x_1 + 4$, $3x_2 + 4$ and $3x_3 + 4$ is

(A) 243 (B) 81

(C) 729 (D) 9

(E) 733

Ans: C

$\text{Var}(x) = 81$

$\text{Var}(3x+4) = 9x\,\text{var}(x)$

$= 9 \times 81$

$= 729$

33. If the median of the observations $4, 6, 7, x, x+2, 12, 12, 13$ arranged in an increasing order is 9 , then the variance of these observations is

(A) $\frac{37}{4}$ (B) $\frac{38}{4}$

(C) 8 (D) 9

(E) 10

Ans: A

$4, 6, 7, x, x+2, 12, 12, 13$

Median $= \frac{x+(x+2)}{2} = x+1$

$x+1 = 9$

$x = 8$

$4, 6, 7, 8, 10, 12, 12, 13$

$\Sigma x^2 = 16 + 36 + 49 + 64 + 100 + 144 + 144 + 169$

$= 722$

$E(x) = 4 + 6 + 7 + 8 + 10 + 12 + 13 = 72$

$$\text{Var}(x) = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{722}{8} - \left(\frac{72}{8}\right)^2$$
$$= \frac{722}{8} - \frac{5184}{64}$$
$$= \frac{592}{64}$$
$$= \frac{37}{4}$$

34. Let $\bar{x}$ denote the mean of the observations $1, 3, 5, a, 9$ and $\bar{y}$ denote the mean of the observations $2, 4, b, 6, 8$ where $a, b > 0$. If $\bar{x} = \bar{y}$, the value of $2(a-b)$ is

(A) 2 (B) 38

(C) 8 (D) -4

(E) 4

Ans: E

$x : 1, 3, 5, a, 9$

$$\bar{x} = \frac{1+3+5+a+9}{5} = \frac{a+18}{5}$$

$y : 2, 4, b, 6, 8$

$$\bar{y} = \frac{2+4+b+6+8}{5} = \frac{b+20}{5}$$

$\bar{x} = \bar{y}$

$$\frac{a+18}{5} = \frac{b+20}{5}$$

$a - b = 20 - 18$

$= 2$

$2(a-b) = 2 \times 2$

$= 4$

35. Consider two independent events E and F such that $P(E) = \frac{1}{4}, P(E \cup F) = \frac{2}{5}$ and $P(F) = a$. Then, the value of a is

(A) $\frac{13}{20}$ (B) $\frac{1}{20}$

(C) $\frac{1}{4}$ (D) $\frac{1}{5}$

(E) $\frac{3}{5}$

Ans: D

$$P(E \cup F) = \frac{2}{5}$$
$$P(E) = \frac{1}{4}$$
$$P(F) = a$$
$$P(E \cup F) = P(E) + P(F) - P(E)P(F)$$
$$\frac{2}{5} = \frac{1}{4} + a - \frac{1}{4}a$$
$$\frac{2}{5} - \frac{1}{4} = \frac{3}{4}a$$
$$\frac{3}{20} = \frac{3}{4}a$$
$$a = \frac{1}{5}$$

36. There are two cash counters A and B for placing orders in a college canteen. Let E_A be the event that there is a queue at counter A and E_B denotes the event that

there is a queue at counter B. If $P(E_A) = 0.45, P(E_B) = 0.55$ and $P(E_A \cap E_B) = 0.25$. then the probability that there is no queue at both the counters is

(A) 0.75 (B) 0.15

(C) 0.25 (D) 0.20

(E) 1.75

Ans: C

$P(E_A) = 0.45$

$P(E_B) = 0.55$

$P(E_A \cap E_B) = 0.25$

$P(E'_A \cap E'_B) = 1 - P(E_A \cup E_B)$

$= 1 - [P(E_A) + P(E_B) - P(E_A \cap E_B)]$

$= 1 - [0.45 + 0.55 - 0.25]$

$= 1 - 0.75$

$= 0.25$

37. Let $S = \{a, b, c\}$ be the sample space with the associated probabilities satisfying $P(a) = 2P(b)$ and $P(b) = 2P(c)$. Then the value of $P(a)$ is

(A) $\frac{1}{5}$ (B) $\frac{2}{7}$

(C) $\frac{1}{7}$ (D) $\frac{1}{6}$

(E) $\frac{4}{7}$

Ans:E

$P(a) = 2P(b), P(b) = 2P(e)$

$P(a) + P(b) + P(c) = 1$

$$P(a) + \frac{P(a)}{2} + \frac{P(b)}{2} = 1$$

$$P(a) + \frac{P(a)}{2} + \frac{P(a)}{4} = 1$$

$$P(a)\left(1 + \frac{1}{2} + \frac{1}{4}\right) = 1$$

$$P(a)\left(\frac{4+2+1}{4}\right) = 1$$

$$P(a)\left(\frac{7}{4}\right) = 1$$

$$P(a) = \frac{4}{7}$$

38. A coin is tossed thrice. The probability of getting a head on the second toss given that a tail has occurred in at least two tosses is

(A) $\frac{1}{2}$ (B) $\frac{1}{16}$

(C) $\frac{1}{8}$ (D) $\frac{1}{4}$

(E) $\frac{1}{3}$

Ans: D

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH$

$TTT\}$

$E = \{HHH, HHT, THH, THT\}$

$P = \{HTT, THT, TTH, TTH\}$

$E \cap P = \{THT\}$

$P(E/P) = 1/4$

39. Let X be a random variable following Binomial distribution; $B\,\text{lin}(n, p)$, where n is the number of independent Bernoulli trials and p is the probability of success. If $E(X) = 1$ and $\text{Var}(X) = \frac{4}{5}$, then the values of n and p are

(A) $n = 5, p = \frac{4}{5}$ (B) $n = 1, p = \frac{1}{5}$

(C) $n = 1, p = 1$ (D) $n = 5, p = \frac{1}{5}$

(E) $n = 1, p = \frac{4}{5}$

Ans: D

$E(x) = 1, \quad \text{var}(x) = \dfrac{4}{5}$

$np = 1, \quad npq = \dfrac{4}{5}$

$\Rightarrow 1 \times q = \dfrac{4}{5}$

$q = \dfrac{4}{5}$

$p = 1 - q = 1 - \dfrac{4}{5} = \dfrac{1}{5}$

$np = 1$

$n \times \dfrac{1}{5} = 1$

$n = 5$

$p = 1/5, n = 5$

40. A box contains 10 coupons, labelled as $1, 2, \ldots 10$. Three coupons are drawn at random and without replacement. Let X_1, X_2 and X_3 denote the numbers on the coupons. Then the probability that $\max\{X_1, X_2, X_3\} < 7$ is

(A) $\dfrac{{}^3\mathrm{C}_1}{{}^{10}\mathrm{C}_3}$ (B) $\dfrac{{}^7C_3}{{}^{10}C_3}$

(C) $\dfrac{{}^3\mathrm{C}_3}{{}^{10}\mathrm{C}_3}$ (D) $\dfrac{{}^3\mathrm{C}_1}{{}^{10}\mathrm{C}_7}$

(E) $\dfrac{{}^6C_3}{{}^{10}C_3}$

Ans: E Total $= \frac{{}^6C_3}{{}^{10}c_3}$

41. An electric bulb manufacturing company manufactures three types of electric bulbs A, B and C. In a room containing these three types of electric bulbs, it is known that 6% of type A electric bulbs are defective, 4% of type B electric bulbs are defective and 2% of type C electric bulbs are defective. An electric bulb is selected at random from a lot containing 50 type A electric bulbs, 30 type B electric bulbs and 20 type C electric bulbs. The selected electric bulb is found to be defective. Then the probability that the selected electric bulb was type A is

(A) $\dfrac{2}{23}$ (B) $\dfrac{23}{500}$

(C) $\dfrac{12}{23}$ (D) $\dfrac{15}{23}$

(E) $\dfrac{6}{115}$

Ans:D

$P(A/E) = \frac{P(A)P(E/A)}{P(A)P(E/A)+P(B)P(E/B)+P(C)P(E/C)}$

$= \dfrac{\frac{1}{2} \times \frac{6}{100}}{\frac{1}{2} \times \frac{6}{100} + \frac{3}{10} \times \frac{4}{100} + \frac{1}{5} \times \frac{2}{100}}$

$= \dfrac{\dfrac{6}{200}}{\dfrac{6 \times 5}{200 \times 5} + \frac{12}{1000} + \frac{2\times2}{500\times2}}$

$= \frac{\frac{6}{200}}{\frac{46}{1000}} = \frac{6}{200} \times \dfrac{1000}{46}$

$= \frac{15}{23}$

42. For four observations x_1, x_2, x_3, x_4, it is given that $\sum_{i=1}^{4} x_i^2 = 656$ and $\displaystyle\sum_{i=1}^{4} x_i = 32$. Then, the variance of these four observations is

(A) 144 (B) 730

(C) 120 (D) 248

(E) 182.5

Ans:

$$\text{Variance} = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$
$$= \frac{656}{4} - \left(\frac{32}{4}\right)^2$$
$$= 164 - 64$$
$$= 100$$

43. An urn contains 8 black marbles and 4 white marbles. Two marbles are chosen at random and without replacement. Then the probability that both marbles are black is

(A) $\dfrac{7}{33}$ (B) $\dfrac{2}{3}$

(C) $\dfrac{7}{11}$ (D) $\dfrac{14}{33}$

(E) $\dfrac{21}{143}$

Ans:D

8 Black 4-white

Total $= {}^{12c_2}$

Probability $= \frac{{}^8C_2}{12c_2} = \frac{\frac{7.8}{1.2}}{\frac{12\times11}{1.2}} = \frac{56}{132} = \frac{14}{33}$

44. A box contains 100 tickets numbered $00, 01, 02, \ldots 99$ and a ticket is drawn at random. Let X denote the sum of the digits on that ticket and Y denote the product of those digits. Then the value of $P(X = 2 \mid Y = 0)$ is

(A) $\frac{3}{19}$ (B) $\frac{6}{19}$

(C) $\frac{1}{19}$ (D) $\frac{2}{19}$

(E) $\frac{1}{100}$

Ans: D

$P(x = 2/Y = 0) = \frac{P(X\cap Y)}{P(Y)} = \frac{\frac{2}{100}}{\frac{19}{100}} = \frac{2}{19}$

45. Let the coefficient of variation of two datasets be 50 and 75 . respectively and the corresponding variances be 25 and 36 . respectively. Also let $\bar{x}_1$ and $\bar{x}_2$ denote the corresponding sample means. Then $\bar{x}_1 + \bar{x}_2$ is

(A) 2 (B) 10

(C) 18 (D) 20

(E) 16

Ans:C

coefficent of viriation = 50

$$\sigma_1^2 = 25$$
$$\sigma_2^2 = 36$$

$$c \cdot V = 75$$

$\frac{\sigma_1}{\bar{x}_1} \times 100 = 50$

$\frac{5 \times 100}{\bar{x}_1} = 50$

$\bar{x}_1 = \frac{5 \times 100}{50} = 10$

$\frac{\sigma_2}{\bar{x_2}} \times 100 = 75$

$\frac{6 \times 100}{\bar{x_2}} = 75$

$\bar{x_2} = \frac{6 \times 100}{75} = 8$

$\bar{x}_1 + \overline{x_2} = 10 + 8 = 18$

46. The mean deviation about the median for the data $3, 5, 9, 3, 8, 10, 7$ is

(A) $\frac{23}{7}$ (B) $\frac{4}{7}$

(C) $-\frac{4}{7}$ (D) $\frac{16}{7}$

(E) $\frac{17}{7}$

Ans:D

we arrange accending order

$3, 3, 5, 7, 8, 9, 10$

Median $= 7$

x	$\lvert x - M\rvert$
3	4
3	4
5	2
7	0
8	1
9	2
10	3
45	16

Mean deviation $= \frac{\sum \lvert x - m\rvert}{n}$

$= \frac{16}{7}$

47. A biased die is rolled such that the probability of getting k dots, $1 \le k \le 6$, on the upper face of the die is proportional to k. Then the probability that five dots appear on the upper face of the die is

(A) $\frac{16}{21}$ (B) $\frac{2}{21}$

(C) $\frac{1}{21}$ (D) $\frac{3}{-21}$

(E) $\frac{5}{21}$

Ans: E

Since the probability of the Faces are proportional to the dots on them we can take the probabilities of faces $1, 2, 3, \ldots .6$ as K, 2k, 3k, ... 6k

we have $k + 2k + \cdots + 6k = 1$

$$21k = 1$$
$$k = \tfrac{1}{21}$$

Probability of (5 dots) $= 5k = \frac{5}{21}$

48. Let $\Omega = \{1, 2, 3, 4, 5\}$ be the sample space with the events $A = \{1, 2, 5\}$, $B = \{1, 3, 5\}$ and $C = \{2, 3, 5\}$. Let E^c denote the complement of an event E. Then $P((A \cap B)^c \cup C^c)$ is

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$

(C) $\frac{2}{5}$ (D) $\frac{4}{5}$

(E) 1

Ans: D

$A \cap B = \{1, 5\}$

$(A \cap B)^c = \{2, 3, 4\}, \quad C^c = \{1, 4\}$

$(A \cap B)^c \cup C^c = \{1, 2, 3, 4\}$

$P((A \cap B)^c \cup C^c) = \frac{4}{5}$

49. For any real number x, the least value of $4 \cos x - 3 \sin x + 5$ is

(A) 10 (B) 2

(C) 0 (D) 8

(E) 4

Ans: C

$$-\sqrt{4^2 + 3^2} \le 4 \cos x - 3 \sin x \le \sqrt{4^2 + 3^2}$$
$$-5 \le 4 \cos x - 3 \sin x \le 5$$
$$-5 + 5 \le 4 \cos x - 3 \sin x + 5 \le 5 + 5$$
$$0 \le 4 \cos x - 3 \sin x + 5 \le 10$$

$\therefore$ least value of $4 \cos x - 3 \sin x + 5$ is 0 .

50. Let $P(x) = \cos^2 x + \sin^4 x$, for any $x \in \mathbb{R}$. Then which of the following options is correct for all x ?

(A) $\frac{1}{6} \le P(x) \le \frac{3}{4}$ (B) $0 \le P(x) \le \frac{1}{2}$

(C) $0 \le P(x) \le 1$ (D) $\frac{1}{2} \le P(x) \le \frac{3}{2}$

(E) $\frac{3}{4} \le P(x) \le 1$

Ans: E

$P(x) = \cos^2 x + \sin^4 x$

$P(x) = 1 - \sin^2 x + \sin^4 x$

$= 1 - \sin^2 x \left(1 - \sin^2 x\right)$

$= 1 - \sin^2 x \cos^2 x$

$= 1 - (\sin x \cos x)^2$

$= 1 - \left(\frac{\sin 2x}{2}\right)^2$

$= 1 - \frac{1}{4} \sin^2 2x \qquad 0 \le \sin^2 2x \le 1$

$1 - \frac{1}{4} \le P(x) \le 1$

$\frac{3}{4} \le P(x) \le 1$

51. Let α and β be such that $\alpha + \beta = \pi$. If $\cos \alpha = \frac{1}{\sqrt{2}}$, then the value of $\cot(\beta - \alpha)$ is

(A) ∞ (B) 1

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

(E) 0

Ans:E

Given $\quad \alpha + \beta = \pi, \quad \cos \alpha = \frac{1}{\sqrt{2}}$

$$\therefore \alpha = \frac{\pi}{4}$$

$$\therefore \beta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\begin{aligned}\cot(\beta-\alpha) &= \cot\left(\frac{3\pi}{4} - \frac{\pi}{4}\right)\\ &= \cot\left(\frac{2\pi}{4}\right)\\ &= \cot\left(\frac{\pi}{2}\right)\\ &= 0\end{aligned}$$

52. The value of $\operatorname{cosec} 20^\circ \tan 60^\circ - \sec 20^\circ$ is

(A) 0　　(B) 1

(C) 2　　(D) 4

(E) 6

Ans:D

$\operatorname{cosec} 20^\circ \tan 60^\circ - \sec 20^\circ$

$= \frac{1}{\sin 20^\circ} \times \sqrt{3} - \frac{1}{\cos 20^\circ}$

$= \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ}$

$= \frac{\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ}{\frac{(\sin 20^\circ \cos 20^\circ)}{2}}$

$= \frac{\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ}{\frac{\sin 40^\circ}{4}}$

$= \frac{4\left(\sin\left(60 - 20^\circ\right)\right)}{\sin 40^\circ} = 4\frac{\sin 40^\circ}{\sin 40^\circ}$

$= 4$

53. If $\alpha + \beta + \gamma = 2\pi$, then the value of $\cot\frac{\alpha}{2}\cot\frac{\beta}{2} + \cot\frac{\alpha}{2}\cot\frac{\gamma}{2} + \cot\frac{\beta}{2}\cot\frac{\gamma}{2}$ is

(A) 0　　(B) 1

(C) $\frac{\pi}{2}$　　(D) $\frac{\pi}{3}$

(E) $\frac{1}{2}$

Ans:B

Given,

$$\alpha + \beta + \gamma = 2\pi$$

$$\frac{\alpha + \beta + \gamma}{2} = \pi$$

$$\frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \pi$$

Let $\frac{\alpha}{2} = A, \quad \frac{\beta}{2} = B, \quad \frac{\gamma}{2} = C$

$$\therefore A + B + C = \pi$$

$$\therefore \quad \cot\frac{\alpha}{2}\cot\frac{\beta}{2} + \cot\frac{\alpha}{2}\cot\frac{\gamma}{2} + \cot\frac{\beta}{2}\cot\frac{\gamma}{2} = 1$$

$$(\cot A \cot B + \cot A \cot C + \cot B \cot C = 1)$$

$$\text{if } A + B + C = \pi$$

54. Let p, q and r be real numbers such that $|r| > \sqrt{p^2 + q^2}$. Then the equation $p\cos\theta + q\sin\theta = r$ has

(A) exactly one real solution.

(B) exactly two real solutions.

(C) infinite number of real solutions.

(D) no real solution.

(E) integer solutions.

Ans:D

Given, $|r| > \sqrt{p^2 + q^2}$

$$-\sqrt{p^2+q^2} \le p\cos\theta + q\sin\theta \le \sqrt{p^2+q^2}$$

$$-\sqrt{p^2+q^2} \le r \le \sqrt{p^2+q^2}$$

We know that, $|r| > \sqrt{p^2 + q^2}$

$\therefore$ No real solution

55. If $x \in (0, \pi)$ satisfies the equation $6^{1+\sin x + \sin^2 x + \cdots} = 36$, then the value of x is

(A) 0　　(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$　　(D) $\frac{\pi}{2}$

(E) $\frac{\pi}{4}$

Ans:C

$6^{1+\sin x + \sin^2 x + \cdots} = 36$

$6^{1+\sin x + \sin^2 x + \cdots} = 6^2$

$$\therefore \quad 1 + \sin x + \sin^2 x + \cdots = 2$$

It is infinite G.P.

$a = 1, r = \sin x \quad -1 \le \sin x \le 1$

$\therefore r \le 1$

Sum of $G \cdot P = \frac{a}{1-r}$
$\therefore \frac{1}{1-\sin x} = 2$
$1 - \sin x = \frac{1}{2}$
$\sin x = \frac{1}{2}$
$\therefore x = \frac{\pi}{6}$

56. The value(s) of $a(\neq 0)$ for which the equation $\frac{1}{2}(x-2)^2 + 1 = \sin\left(\frac{a}{x}\right)$ holds is/are

(A) $(4n+1)\pi, n \in \mathbb{Z}$

(B) $2(n-1)\pi, n \in \mathbb{Z}$

(C) $n\pi, n \in \mathbb{N}$

(D) $\frac{n\pi}{2}, n \in \mathbb{N}$

(E) 1

Ans:A

$\frac{1}{2}(x-2)^2 + 1 = \sin\left(\frac{a}{x}\right)$
$\frac{1}{2}(x-2)^2 + 1 \geq 1$
$-1 \leq \sin\left(\frac{a}{x}\right) \leq 1$

$$\therefore \quad \sin\left(\frac{a}{x}\right) = 1$$

$$\text{At } x = 2, \quad \frac{1}{2}(x-2)^2 + 1 = 1$$

$$\therefore \quad \sin\left(\frac{a}{2}\right) = 1$$

$$\frac{a}{2} = (4n+1)\frac{\pi}{2}$$

$$\therefore a = (4n+1)\pi, n \in Z$$

57. If x is a real number such that $\tan x + \cot x = 2$, then $x =$

(A) $\left(n + \frac{1}{4}\right)\pi, n \in \mathbb{Z}$

(B) $(n+1)\pi, n \in \mathbb{Z}$

(C) $\left(n + \frac{1}{2}\right)\pi, n \in \mathbb{Z}$

(D) $n\pi, n \in \mathbb{Z}$

(E) $\frac{2}{3}n\pi, n \in \mathbb{Z}$

Ans:A

$$\tan x + \cot x = 2$$

$$\tan x + \frac{1}{\tan x} = 2$$

$$\frac{\tan^2 x + 1}{\tan x} = 2$$

$$\tan^2 x + 1 = 2\tan x$$

$$\tan^2 x - 2\tan x + 1 = 0$$

$$(\tan x - 1)^2 = 0$$

$$\tan x - 1 = 0$$

$$\tan x = 1$$

$$\therefore x = \left(n + \tfrac{1}{4}\right)\pi, n \in z$$

58. If $\frac{1+\sin x}{1-\sin x} = \frac{(1+\sin y)^3}{(1-\sin y)^3}$ for some real values x and y, then $\frac{\sin x}{\sin y} =$

(A) $\frac{3+\sin^2 y}{1+3\sin^2 y}$

(B) $\frac{3+\cos^2 y}{1+3\cos^2 y}$

(C) $\frac{3+\sin^2 y}{1-3\sin^2 y}$

(D) $\frac{3+\sin^2 y}{1-3\cos^2 y}$

(E) $\frac{1+3\sin^2 y}{1-3\cos^2 y}$

Ans:A

$\frac{1+\sin x}{1-\sin x} = \frac{(1+\sin y)^3}{(1-\sin y)^3}$

$\frac{1+\sin x}{1-\sin x} = \frac{1+\sin^3 y + 3\sin y + 3\sin^2 y}{1-\sin^3 y + 3\sin^2 y - 3\sin y}$

Use componendo-dividendo rule,

$\frac{1+\sin x+1-\sin x}{1+\sin x-(1-\sin x)}$
$= \frac{1+\sin^3 y+3\sin y+3\sin^2 y+1-\sin^3 y+3\sin^2 y-3\sin y}{1+\sin^3 y+3\sin y+3\sin^2 y-\left(1-\sin^3 y+3\sin^2 y-3\sin y\right)}$

$\frac{2}{2\sin x} = \frac{2+6\sin^2 y}{2\sin^3 y + 6\sin y}$

$\frac{1}{\sin x} = \frac{1+3\sin^2 y}{\sin^3 y + 3\sin y}$

$\sin x = \frac{\sin^3 y + 3\sin y}{1+3\sin^2 y}$

$\frac{\sin x}{\sin y} = \frac{\sin^2 y + 3}{1+3\sin^2 y}$

59. Let k be a real number such that $\sin\frac{3\pi}{14}\cos\frac{3\pi}{14} = k\cos\frac{\pi}{14}$. Then the value of $4k$ is

(A) 1

(B) 2

(C) 3 (D) 4

(E) 0

Ans:B

$\sin\frac{3\pi}{14}\cos\frac{3\pi}{14} = k\cos\frac{\pi}{14}$

$2\sin\frac{3\pi}{14}\cos\frac{3\pi}{14} = 2k\cos\frac{\pi}{14}$

$\sin\frac{6\pi}{14} = 2k\cos\frac{\pi}{14}$

$\left(\therefore \frac{6\pi}{14}+\frac{\pi}{14}=\frac{7\pi}{14} \quad =\frac{\pi}{2}\right)$

$\therefore \sin\frac{6\pi}{14} = \cos\frac{\pi}{14}$

$1 = 2k$

$\therefore \quad k = \frac{1}{2}$

$\therefore 4k = 4\times\frac{1}{2}$

$4k = 2$

60. In a triangle ABC, if $\cos^2 A - \sin^2 B + \cos^2 C = 0$, then the value of $\cos A\cos B\cos C$ is

(A) $\frac{1}{4}$ (B) 1

(C) $\frac{\pi}{2}$ (D) $\frac{1}{2}$

(E) 0

Ans: E

$\triangle ABC, \quad A+B+C=\pi$

$\cos A\cos B\cos C$

$= \cos(\pi-(B+C))\cdot\cos B\cdot\cos C$

$= -\cos(B+C)\cos B\cos C$

$= -\frac{1}{2}[\cos(B+C+B)+\cos(B+C-B)]\cos C$

$= -\frac{1}{2}[\cos(2B+C)+\cos C]\cos C$

$= -\frac{1}{2}[\cos(2B+(\pi-(A+B))\times \cos(\pi-(A+B))+\cos^2 C]$

$= -\frac{1}{2}[\cos(\pi-(A-B))\cos(\pi-(A+B)) + \cos^2 C]$

$= -\frac{1}{2}[-\cos(A-B)\cdot(-\cos(A+B)) + \cos^2 C]$

$= -\frac{1}{2}[\cos(A-B)\cdot\cos(A+B)+\cos^2 C]$

$= -\frac{1}{2}[\cos^2 A - \sin^2 B + \cos^2 C]$

$= -\frac{1}{2}\times 0$

$= 0$

$\left(\text{ given } \quad \cos^2 A - \sin^2 B + \cos^2 C = 0\right)$

61. The value of $\cos^{-1}\left(\cos\left(\frac{7\pi}{4}\right)\right)$ is

(A) 0 (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

(E) $\frac{\pi}{6}$

Ans:D

$\cos^{-1}(\cos 7\pi/4) = \cos^{-1}\left(\cos\left(2\pi-\frac{7\pi}{4}\right)\right]$

$= \cos^{-1}(\cos\pi/4)$

$= \frac{\pi}{4}$

62. The value of $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{2}{5}\right)$ is

(A) $\tan^{-1}(5)$ (B) $\tan^{-1}\left(\frac{1}{5}\right)$

(C) $\tan^{-1}\left(\frac{2}{3}\right)$ (D) $\text{tin}^{-1}\left(\frac{8}{9}\right)$

(E) $\tan^{-1}\left(\frac{9}{8}\right)$

Ans:E

$\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{2}{5}\right)$

$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{2}{5}}{1-\frac{1}{2}\times\frac{2}{5}}\right)$

$= \tan^{-1}\left(\frac{\frac{9}{10}}{1-1/5}\right)$

$= \tan^{-1}\left(\frac{\frac{9}{10}}{4/5}\right)$

$= \tan^{-1}\left(\frac{9}{8}\right)$

63. The value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

(E) $\frac{\pi}{6}$

Ans:E

$\tan^{-1}(\sqrt{3}) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

$= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

64. Let $\vec{a} = i - j + 2\hat{k}$. Then the vector in the direction of $\hat{a}$ with magnitude 5 units is

(A) $5\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$

(B) $-5\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$

(C) $\frac{1}{\sqrt{6}}(5i - 5j + 10\hat{k})$

(D) $\frac{1}{\sqrt{6}}(-5i - 5j + 10\hat{k})$

(E) $\frac{1}{\sqrt{6}}(10i - 5j + 5\hat{k})$

Ans:C

$\vec{a} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\hat{a} = \frac{\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{6}}$

$\vec{V} = 5\vec{a} = \frac{5}{\sqrt{6}}(\hat{\imath} - \hat{\jmath} + 2\hat{k})$

65. Let $\vec{a} = i + j + 2\hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ be two vectors. Then the unit vector in the direction of $\vec{a} - \vec{b}$ is

(A) $\frac{1}{\sqrt{10}}(2\hat{\jmath} - 3\hat{k})$ (B) $\frac{1}{\sqrt{10}}(3\hat{\jmath} - \hat{k})$

(c) $3j - \hat{k}$ (D) $\frac{1}{\sqrt{5}}(2j - 3\hat{k})$

(E) $\frac{-1}{\sqrt{5}}(2j - 3\hat{k})$

Ans:B

$$\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$$
$$\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$$
$$\vec{a} - \vec{b} = 3\hat{\jmath} - \hat{k}$$
$$|\vec{a} - \vec{b}| = \sqrt{3^2 + 1^2} = \sqrt{10}$$

unit vector in the direction of $\vec{a} - \vec{b}$

$$= \frac{3\hat{\jmath} - \hat{k}}{\sqrt{10}}$$

66. The direction cosines of the vector $\bar{a} = -2\hat{\imath} + \hat{\jmath} - \hat{k}$ are

(A) $\left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ (B) $\left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

(C) $\left(\frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$ (D) $\left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

(E) $\left(\frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

Ans:B

(B)

$$\vec{a} = -2\hat{\imath} + \hat{\jmath} - \hat{k}$$
$$|\vec{a}| = \sqrt{(-2)^2 + \hat{1}^2 + (-1)^2}$$
$$= \sqrt{6}$$

Direction cosine of $\vec{a}$

$$= \left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$$

67. The value of λ for which the vactors $\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\lambda\hat{\imath} + 3\hat{\jmath} + \hat{k}$ are perpendicular is

(A) -2 (B) 2

(C) 0 (D) 1

(E) -1

Ans:A

let $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$

$$\vec{b} = \lambda\hat{\imath} + \hat{\jmath} - \hat{k}$$

if $\vec{a}$ and $\vec{b}$ are perpendicular, then

$$\vec{a} \cdot \vec{b} = 0.$$
$$\vec{a} \cdot \vec{b} = 0 \Rightarrow \lambda + 1 + 1 = 0$$
$$\Rightarrow \lambda = -2$$

68. The position vectors of two poins P and Q

are given by $\overrightarrow{OP} = 2\vec{a} - \vec{b}$ and $\overrightarrow{OQ} = \vec{a} + 3\vec{b}$, respectively. If a point R divides the line joining P and Q internally in the ratio $1:2$, then the position vector of the point R is

(A) $\frac{1}{3}(5\vec{a} - \vec{b})$ (B) $\frac{1}{3}(5\vec{a} + \vec{b})$
(C) $\frac{1}{3}(\vec{a} - 5\vec{b})$ (D) $\frac{1}{3}(\vec{a} + 5\vec{b})$
(E) $\frac{1}{3}(\vec{a} + \vec{b})$

Ans:B

$\overrightarrow{OP} = 2\vec{a} - \vec{b}$

$\overrightarrow{OQ} = \vec{a} + 3\vec{b}$

$$\overrightarrow{OR} = \frac{\overrightarrow{OQ} + 2\overrightarrow{OP}}{3} = \frac{\vec{a} + 3\vec{b} + 2(2\vec{a} - \vec{b})}{3} = \frac{5\vec{a} + \vec{b}}{3}$$

69. Let $\vec{a}$ and $\vec{b}$ be perpendicular vectors such that $|\vec{a}| = \sqrt{104}$ and $|\vec{b}| = 6$. Then the value of $|\vec{a} - \vec{b}|$ is

(A) $\sqrt{110}$ (B) $\sqrt{140}$
(C) $\sqrt{98}$ (D) $\sqrt{55}$
(E) $\sqrt{70}$

Ans:B

$$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$
$$= 104 + 36$$
$$= 140$$
$$|\vec{a} - \vec{b}| = \sqrt{140}$$

70. Let x be a real number and $\vec{a}$ be any non-zero vector such that $|(4 - x)\vec{a}| < |3\vec{a}|$ Then which of the following options is correct ?

(A) $0 < x < 6$ (B) $0 < x < 7$
(C) $1 < x < 7$ (D) $1 \le x \le 7$
(E) $0 \le x \le 6$

Ans: C

$|(4 - x)\vec{a}| < |3\vec{a}|$

$|4 - x| < 3$

$-3 < 4 - x < 3$

$-3 < x - 4 < 3$

$1 < x < 7$

71. The value of λ for which the vectors $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + \lambda\hat{\jmath} - 8\hat{k}$ are collinear is

(A) 0 (B) 1
(C) 3 (D) 6
(E) 4

Ans:D

$\overrightarrow{a_1} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$\overrightarrow{a_2} = -4\hat{\imath} + \lambda\hat{\jmath} - 8\hat{k}$

$$\frac{2}{-4} = \frac{-3}{\lambda} = \frac{4}{-8}$$
$$\frac{-1}{2} = \frac{-3}{\lambda} = \frac{-1}{2}$$
$$\lambda = 6$$

72. The projection of the vector $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ is

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$
(C) $\frac{2}{3}$ (D) $\frac{1}{3}$
(E) 0

Ans:B

$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$$\hat{b} = \frac{\hat{\imath} + 2\hat{\jmath} + 2\hat{k}}{\sqrt{9}}$$
$$\vec{a} \cdot \hat{b} = \frac{2 - 6 + 8}{3} = \frac{4}{3}$$

73. Let $f(x) = \begin{cases} -5, & x \le 0 \\ x - 5, & x > 0 \end{cases}$ and $g(x) = |f(x)| + 2f(|x|)$. Then $g(-2)$ will be

(A) -1 (B) -15
(C) 1 (D) 0
(E) -11

Ans:A

$$f(x)=\begin{cases}-5, & x\leqslant 0\\ x-5, & x>0\end{cases}$$

$g(x)=|f(x)|+2f(|x|)$

$g(-2)=|f(2)|+2f(|-2|)$

$=|-5|+2f\,(2)$

$=5+2(2-5)$

$=-1$

74. Let [.] denote the greatest integer function and $f(x)=[x]+|2-x|, -1\leq x\leq 4$. Then

(A) f is continuous at $x=2$.

(B) f is not continuous at $x=1$.

(C) f is continuous at $x=0$.

(D) f is differentiable at $x=3$.

(E) f is not differentiable at $x=\frac{3}{2}$

Ans:B

$f(x)=[x]+|2-x|, \quad -1\leq x\leq 4$ f is not continuous at $x=1$ Because $[x]$ is not continuous at $x=1$ [Greatest integer function is continuous at all points except integer points]

75. $\lim_{x\to 0}\dfrac{e^x-1}{3\left(1-e^{2x}\right)}=$

(A) $\dfrac{1}{6}$ (B) $-\dfrac{1}{6}$

(C) 3 (D) 0

(E) $-\dfrac{1}{3}$

Ans:B

$\lim_{x\to 0}\dfrac{e^x-1}{3\left(1-e^{2x}\right)}=\lim_{x\to 0}\dfrac{e^x}{3\left(-2e^{2x}\right)}$

$=\dfrac{1}{3\cdot -2}$

$=-\dfrac{1}{6}$

76. Let $f(x)=\left(1-\frac{1}{x}\right)^2, x>0$. Then

(A) f is increasing in $(0,2)$ and decreasing in $(2,\infty)$.

(B) f is decreasing in $(0,2)$ and increasing in $(2,\infty)$.

(C) f is increasing in $(0,1)$ and decreasing in $(1,\infty)$.

(D) f is decreasing in $(0,1)$ and increasing in $(1,\infty)$.

(E) f is increasing in $(0,\infty)$.

Ans:D

$f(x)=\left(1-\dfrac{1}{x}\right)^2, \quad x>0$

$f'(x)>0$

$f'(x)=2\left(1-\dfrac{1}{x}\right)\times\dfrac{1}{x^2}>0$

$=\left(1-\dfrac{1}{x}\right)\times\dfrac{1}{x^2}>0$

$1-\dfrac{1}{x}>0, because\ x^2>0$

$1>\dfrac{1}{x},$

$x>1$

f is increasing in $(1,\infty)$ and f is decreasing in $(-\infty,1)$. But $x>0$ $\therefore f$ is increasing in $(1,\infty)$ and decreasing in $(0,1)$

77. Let $f:R\to R$ be defined by

$$f(x)=\begin{cases}3e^x & \text{if} \quad x<0\\ x^2+3x+3 & \text{if} \quad 0\leq x<1\\ x^2-3x-3 & \text{if} \quad x\geq 1\end{cases}$$

(A) f is continuous on R.

(B) f is not continuous on R.

(C) f is continuous on R $\{0\}$

(D) f is continuous on R $\{1\}$

(E) f is not continuous on R $\{0,1\}$

Ans:D

$$f\left(0^-\right)=3$$
$$f\left(0^+\right)=3=f(0)$$

$\therefore f$ is continuous at $x=0$

$$f(1^-)=7, f(1^+)=-5,$$

f is discuntinuous at $x=1$ $\therefore f$ is continuous on $R\backslash\{1\}$

78. Let $f(x) = \pi \cos x + x^2$. The value of $c \in (0, \pi)$ where f attains its local maximum / minimum is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{4}$ (D) $\frac{\pi}{3}$

(E) $\frac{\pi}{6}$

Ans:B

$$f(x) = \pi \cos x + x^2$$
$$f'(c) = 0$$
$$\Rightarrow -\pi \sin c + 2c = 0$$

which is satisfied by $c = \pi/2$

79. The minimum of $f(x) = \sqrt{10 - x^2}$ in the interval $[-3, 2]$ is

(A) $\sqrt{4}$ (B) $\sqrt{6}$

(C) 1 (D) 0

(E) $\sqrt{10}$

Ans:C

$f(x) = \sqrt{10 - x^2}$ has minimum when x^2 is maximum

max $x^2 = 9$ in $[-3, 2]$.

$\therefore$ minimum of $f(x) = \sqrt{10 - 9} = \sqrt{1}$
=1

80. The equation of the line passing through origin which is parallel to the tangent of the curve $y = \frac{x-2}{z-3}$ at $x = 4$ is

(A) $y = 2x$ (B) $y = -2x + 1$

(C) $y = -x$ (D) $y = x + 2$

(E) $y = 4x$

Ans:C

$$y = \frac{x-2}{x-3}$$

$x = 4 \Rightarrow y = \frac{2}{1} = 2.$

$\frac{dy}{dx} = \frac{(x-3) - (x-2)}{(x-3)^2} = \frac{-1}{(x-3)^2}$

at $x = 4$. $\frac{dy}{dx} = -1$

tangent is

$$y - 2 = -1(x - 4) \Rightarrow y - 2 = -x + 4$$
$$x + y - 6 = 0$$
$$x + y + k = 0$$
$$x + y = 0$$
$$y = -x$$

81. Let $f(x) = a \sin^2 3x$. If $f'\left(\frac{\pi}{12}\right) = -3$, then the value of α is

(A) -1 (B) -π

(C) π (D) $\frac{\pi}{2}$

(E) 1

Ans:A

$f(x) = \alpha \sin^2 3x$

$f'(x) = 3\alpha \sin 6x$

$f'(\pi/12) = -3 \Rightarrow 3\alpha \sin\left(\frac{\pi}{2}\right) = -3$

$3\alpha = -3$

$\alpha = -1$

82. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 2x + 3, & x \leq 5 \\ 3x + \alpha, & x > 5 \end{cases}$$

Then the value of α so that f is continuous on $\mathbb{R}$ is

(A) 2 (B) -2

(C) 3 (D) -3

(E) 0

Ans:B

$f(x) = \begin{cases} 2x + 3 & , x \leq 5 \\ 3x + \alpha & , x > 5 \end{cases}$

$f\left(5^-\right) = 13 = 15 + \alpha$

$\alpha = 13 - 15 = -2.$

83. If $y = x^{e^x} + x^e$ for $x > 0$, then $\frac{dy}{dx}$ is equal to

(A) $x^{e^x}\left[\frac{1}{x} + \ln x\right] + e^x$

(B) $x^{e^x} e^x \left[\frac{1}{x} + \ln x\right] + ex^{e-1}$

(C) $e^x \cdot x^{e^x - 1} + ex^e$

(D) $x^{e^x}e^{-x}\left[\frac{1}{x}-\ln x\right]+ex^{e-1}$
(E) $x^{e^x}e^{x}\left[\frac{1}{x}-\ln x\right]+ex^{e-1}$
Ans:B
$y=x^{e^x}+x^e$
$$\frac{dy}{dx}=x^{e^x}\left[\frac{e^x}{x}+ex\cdot\ln x\right]+ex^{e-1}$$
$$=x^{e^x}\cdot e^x\left[\frac{1}{x}+\ln x\right]+ex^{e-1}$$

84. $\lim_{x\to 0}\dfrac{\ln(1+(\ln 5)x)}{5^x-1}=$
(A) 1
(B) $\ln 5$
(C) -1
(D) 5
(E) $\dfrac{1}{5}$
Ans:A
$$\lim_{x\to 0}\frac{\ln(1+x\ln 5)}{5^x-1}\quad \frac{0}{0}\text{ form}$$
$$=\lim_{x\to 0}\frac{\frac{1}{1+x\ln 5}\cdot\ln 5}{5^x\ln 5}=\frac{\ln 5}{\ln 5}$$
$$=1$$

85. $\int\dfrac{1}{x^2-2x+2}dx=$
(A) $\tan^{-1}(x-1)+C$
(B) $\sin^{-1}(2x-1)+C$
(C) $\sin^{-1}(x-1)+C$
(D) $\tan^{-1}(2x-1)+C$
(E) $\dfrac{1}{(2x-1)^3}+C$
Ans:A
$$\int\frac{1}{x^2-2x+2}dx=\int\frac{1}{(x-1)^2+1}dx$$
$$=\tan^{-1}(x-1)+C$$

86. $\int\sin^2\pi x dx=$
(A) $\dfrac{x}{2}-\dfrac{1}{4\pi}\sin 2\pi x+C$
(B) $\dfrac{x}{2}+\dfrac{1}{8\pi}\sin 4\pi x+C$
(C) $\dfrac{x}{8}-\dfrac{1}{4\pi}\cos 2\pi x+C$
(D) $x+\dfrac{1}{2\pi}\sin 2\pi x+C$
(E) $\dfrac{x}{2}-\dfrac{1}{2\pi}\cos 2\pi x+C$
Ans:A
$$\int\sin^2\pi x dx=\int\left(\frac{1-\cos 2\pi x}{2}\right)dx$$
$$=\frac{1}{2}\int(1-\cos 2\pi x)dx$$
$$=\frac{1}{2}\left[x-\frac{\sin 2\pi x}{2\pi}\right]+C$$
$$=\frac{x}{2}-\frac{1}{4\pi}\sin 2\pi x+C$$

87. $\int\dfrac{x+5}{x^2-1}dx=$
(A) $3\ln|x-1|-2\ln|x+1|+\bar{C}$
(B) $2\ln|x-1|-3\ln|x+1|+C$
(C) $\ln|x-2|+\ln|x+1|+C$
(D) $\ln|x+2|+\ln|x-1|+C$
(E) $2\ln|x-1|+3\ln|x+1|+C$
Ans:A
$\int\frac{x+5}{x^2-1}dx$
$=\int\frac{x}{x^2-1}dx+5\int\frac{1}{x^2-1}dx$
$$=\frac{1}{2}\int\frac{2x}{x^2-1}dx+5\int\frac{1}{x^2-1}dx$$
$$=\frac{1}{2}\log\left|x^2-1\right|+5\times\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+C$$
$$=\frac{1}{2}\log|(x-1)(x+1)|$$
$$+\frac{5}{2}\log\left|\frac{x-1}{x+1}\right|+C$$
$$=\frac{1}{2}\log|x-1|+\frac{1}{2}\log|x+1|$$
$$+\frac{5}{2}\log|x-1|$$
$$-\frac{5}{2}\log|x+1|+C$$
$$=3\log|x-1|-2\log|x+1|+C$$

88. $\int\dfrac{2\tan x+3}{\sin^2 x+2\cos^2 x}dx=$
(A) $\dfrac{3}{\sqrt{2}}\sin^{-1}\left(\dfrac{\sin x}{\sqrt{2}}\right)+\ln|\sin^2 x+2|+C$
(B) $\dfrac{3}{\sqrt{2}}\tan^{-1}\left(\dfrac{\tan x}{\sqrt{2}}\right)+\ln|\tan^2 x+2|+C$
(C) $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{\tan x}{\sqrt{2}}\right)-\ln|\tan^2 x+2|+C$
(D) $\dfrac{3}{\sqrt{2}}\cos^{-1}\left(\dfrac{\cos x}{\sqrt{2}}\right)+\ln|\sin^2 x+2|+C$
(E) $\dfrac{1}{\sqrt{2}}\cos^{-1}\left(\dfrac{\cos x}{\sqrt{2}}\right)-\ln|\cos^2 x+2|+C$
Ans:B

$= \int \dfrac{2\tan x + 3}{\sin^2 x + 2\cos^2 x} dx =$

dividing numerator and denomenator by $cos^2 x$

$= \displaystyle\int \frac{(2\tan x + 3)\sec^2 x dx}{\tan^3 x + 2}$

$= \displaystyle\int \frac{(2u+3)du}{u^2+2}$

$= \displaystyle\int \frac{2u}{u^2+2} du + 3\int \frac{1}{a^2+2} du$

$= \log\left(u^2+2\right) + 3\dfrac{1}{\sqrt{2}} \tan^{-1}\left(\dfrac{u}{\sqrt{2}}\right) + c$

$= \log\left(\tan^2 x + 2\right) + \frac{3}{\sqrt{2}} \tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + c$

89. $\int x \log\left(1+x^2\right) dx =$

(A) $\dfrac{1}{2}\left(1+x^2\right)\log\left(1+x^2\right) + \frac{x^2}{2} + C$

(B) $\dfrac{1}{2}\left(1+x^2\right)\log\left(1+x^2\right) - \frac{x^2}{2} + C$

(C) $\dfrac{1}{2}\left(1+x^2\right)\log\left(2+x^2\right) - \frac{x^2}{2} + C$

(D) $\left(1+x^2\right)\log\left(1+x^2\right) + \left(1+x^2\right) + C$

(E) $\left(1-x^2\right)\log\left(1+x^2\right) + \left(1-x^2\right) + C$

Ans:B

$\int x \log\left(1+x^2\right) dx$

$= \displaystyle\int \log\left(1+x^2\right) x dx$

$= \log\left(1+x^2\right) \times \dfrac{x^2}{2} - \displaystyle\int \frac{1}{1+x^2}$

$\times 2x \times \dfrac{x^2}{2} dx$

$= \dfrac{x^2}{2} \log\left(1+x^2\right) - \displaystyle\int \frac{x^3}{1+x^2} dx$

$= \dfrac{x^2}{2} \log\left(1+x^2\right) - \displaystyle\int \left(x - \frac{x}{x^2+1}\right) dx$

$= \dfrac{x^2}{2} \log\left(1+x^2\right)$

$- \left[\dfrac{x^2}{2} - \dfrac{1}{2} \log\left(x^2+1\right)\right] + C$

$= \dfrac{x^2}{2} \log\left(1+x^2\right)$

$- \dfrac{x^2}{2} + \dfrac{1}{2} \log\left(x^2+1\right) + C$

$= \dfrac{1}{2}\left(1+x^2\right)\log\left(1+x^2\right) - \dfrac{x^2}{2} + C$

90. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \left\{\begin{array}{ll} x & \text{if } x \le 1 \\ -x+2 & \text{if } x > 1 \end{array}, x, 1+2\right\} = 1$

Then $\int_0^2 f(x) dx =$

(A) $\frac{\pi}{2}$ (B) 1

(C) 2 (D) 4

(E) $\frac{\pi}{6}$

Ans:B

$\displaystyle\int_0^2 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx$

$= \displaystyle\int_0^1 x dx + \int_1^2 (-x+2)dx$

$= \left[\dfrac{x^2}{2}\right]_0^1 + \left[-\dfrac{x^2}{2} + 2x\right]_1^2$

$= \dfrac{1}{2} - 0 + \left[-\dfrac{4}{2} + 4 - \left(-\dfrac{1}{2} + 2\right)\right]$

$= \dfrac{1}{2} - 2 + 4 + \dfrac{1}{2} - 2$

$= 1$

91. $\int \dfrac{1}{\cos x(\sin x + 2\cos x)} dx =$

(A) $\ln|1 - \tan x| + C$

(B) $\ln|3 + \sin x| + C$

(C) $\ln|2 + \tan x| + C$

(D) $\ln|1 + 2\sec x| + C$

(E) $\ln|2 - \tan x| + C$

Ans:C

$\int \frac{1}{\cos x(\sin x + 2\cos x)} dx$ dividing both numerator and denominator by $\cos x$. we get

$= \displaystyle\int \frac{\sec^2 x}{\tan x + 2} dx$

$= \log|\tan x + 2| + c$

92. $\displaystyle\int_0^1 \frac{2e^x}{1+e^{2x}} dx =$

(A) $4\left(\tan^{-1} 2 - \pi\right)$ (B) $2\left(\tan^{-1} e - \frac{\pi}{2}\right)$

(C) $2\left(\tan^{-1} e + \frac{\pi}{4}\right)$ (D) $2\left(\tan^{-1} e - \frac{\pi}{4}\right)$

(E) $2\left(\tan^{-1} 2 + \pi\right)$

Ans:D

$\int_0^1 \frac{2e^x}{1+e^{2x}} dx = 2\int_0^1 \frac{e^x}{1+e^{2x}} dx$

$put,\ t = e^x$

$dt = e^x dx$

$when\ 0,\ t = 1$

$1, t - e$

$= 2\int_1^e \frac{dt}{1+t^2}$
$= 2\left[\tan^{-1} t\right]_1^e$
$= 2\left[\tan^{-1} e - \tan^{-1}(1)\right]$
$= 2\left(\tan^{-1} e - \frac{\pi}{4}\right)$

93. $\int_0^1 \left(5xe^{2x} - \tan\frac{\pi}{4}\right) dx =$

(A) $\dfrac{5}{4}e^2 + \dfrac{1}{4}$ (B) $-\dfrac{5}{4}e^2 - \dfrac{1}{4}$

(C) $\dfrac{5}{4}e^2 - \dfrac{9}{4}$ (D) $\dfrac{3}{4}e^2 + \dfrac{1}{4}$

(E) $\dfrac{1}{4}e^2 + \dfrac{5}{4}$

Ans:A

$$\int_0^1 \left(5xe^{2x} - \tan\frac{\pi}{4}\right) dx$$
$$= 5\int_0^1 xe^{2x}dx - \int_0^1 dx$$
$$= 5\left[\left[x\frac{e^{2x}}{2}\right]_0^1 - \int_0^1 \frac{e^{2x}}{2}dx\right] - (x)_0^1$$
$$= 5\left(\frac{e^2}{2} - 0 - \left(\frac{e^{2x}}{4}\right)_0^1 - 1 - 0\right.$$
$$= 5\left(\frac{e^2}{2} - \frac{e^2}{4} - \frac{1}{4}\right] - 1$$
$$= \frac{5e^2}{2} - \frac{5e^2}{4} + \frac{5}{4} - 1$$
$$= \frac{5e^2}{4} + \frac{1}{4}$$

94. The area of the region in the first quadrant enclosed by the curves $y = \sqrt{x}, y = -x+6$ and the x-axis is

(A) $\dfrac{22}{7}$ (B) $\dfrac{22}{3}$

(C) 12 (D) 24

(E) 8

Ans:B

$y = \sqrt{x}, y = -x + 6$

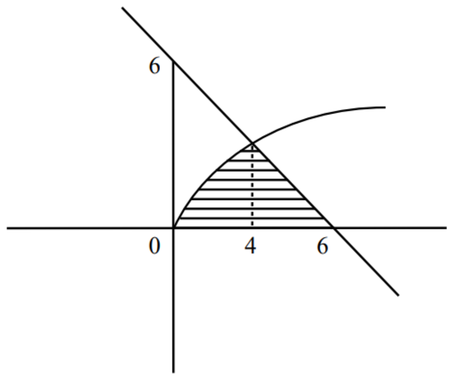
$\sqrt{x} = 6 - x$

$x = 36 - 12x + x^2$

$x^2 - 13x + 36 = 0$

$(x-4)(x-9) = 0$

$x = 4 \quad x = 9$



$$\text{Area } = \int_0 f(x)dx$$
$$= \int_0^6 f(x)dx$$
$$= \int_0^4 \sqrt{x}dx + \int_4^6 (-x+6)dx$$
$$= \frac{2}{3}\left(x^{3/2}\right)_0^4 + \left(-\frac{x^2}{2} + 6x\right)_4^6$$
$$= \frac{2}{3} \times 8 + -18 + 36 - (-8 + 24)$$
$$= \frac{16}{3} + 18 + 8 - 24 = \frac{16}{3} + 2 = \frac{22}{3}$$

95. The area of the region in the first quadrant which is above the parabola $y = x^2$ and enclosed by the circle $x^2 + y^2 = 2$ and the y-axis is

(A) $\dfrac{1}{6} + \dfrac{\pi}{4}$ (B) $\dfrac{1}{12} + \dfrac{\pi}{6}$

(C) $-\dfrac{1}{6} + \dfrac{\pi}{4}$ (D) $\dfrac{1}{4} + \dfrac{\pi}{6}$

(E) $-\dfrac{\pi^2}{2} + 4$

Ans:A

$y = x^2 \quad x^2 + y^2 = 2$

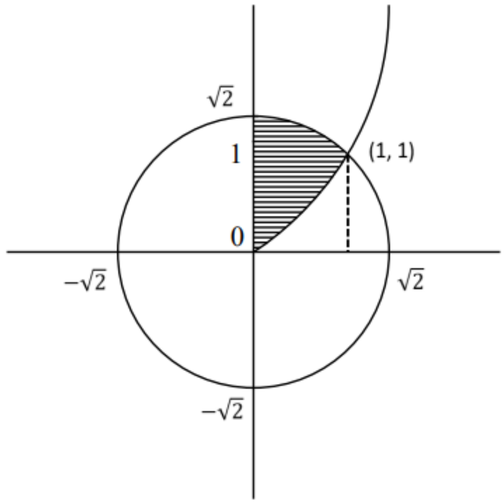
$x^2 + y^2 = 2$

$y + y^2 = 2$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = -2, 1$



$$A=\int_0^1 \sqrt{y}dy+\int_1^{\sqrt{2}}\sqrt{2-y^2}dy$$
$$=\frac{2}{3}\left(y^{3/2}\right)_0^1+$$
$$\left[\frac{ty}{2}\sqrt{2-y^2}+\sin^{-1}\left(\frac{y}{\sqrt{2}}\right)\right]_1^{\sqrt{2}}$$
$$=\frac{2}{3}+0+\frac{\pi}{2}-\left(\frac{1}{2}+\frac{\pi}{4}\right)$$
$$=\frac{2}{3}+\frac{\pi}{2}-\frac{1}{2}-\frac{\pi}{4}$$
$$=\frac{\pi}{4}+\frac{1}{6}$$

96. $\int_0^1 \dfrac{x}{x^3-4}dx=$

(A) $-\dfrac{\pi^2}{6}$ (B) $-\dfrac{22}{7}$

(C) $\ln\left(\dfrac{\sqrt{3}}{2}\right)$ (D) $\ln\left(\dfrac{3}{2}\right)$

(E) $\ln\left(\dfrac{3}{\sqrt{2}}\right)$

Ans:C

$\int_0^1 \frac{x}{x^2-4}dx$

$$=\frac{1}{2}\int\frac{2x}{x^2-4}dx$$
$$=\frac{1}{2}\left[\log\left|x^2-4\right|\right]_0^1$$
$$=\frac{1}{2}\left[\log 3-\log 4\right]$$
$$=\frac{1}{2}\log\frac{3}{4}$$
$$=\log\left(\frac{3}{4}\right)^{1/2}$$
$$=\log\frac{\sqrt{3}}{2}$$

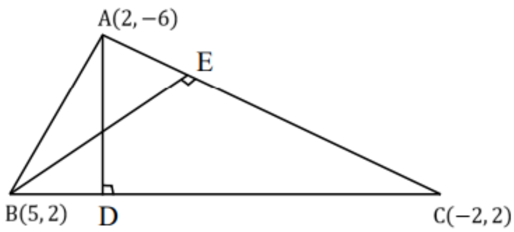
97. If $(2,-6),(5,2)$ and $(-2,2)$ constitute the vertices of a triangle, then the line joining the origin and its orthocentre is

(A) $x+4y=0$ (B) $x-4y=0$

(C) $4x-y=0$ (D) $4x+y=0$

(E) $x-y=0$

Ans:B



slope of $BC=0$

$\therefore$ slope of $AD=\infty$

ie, parallel to y-axis.

$\therefore$ equation is $x=2....(1)$

Slope of $AC=\frac{8}{-4}=-2$ Slope of $BE=\frac{1}{2}$

$\therefore$ The generalize equation of BE by arbitary points

$$y-2=\frac{1}{2}(x-5)$$

$x-2y-1=0....(2)$

Solve Eq (1) and (2)

$$2-2y-1=0$$
$$y=1/2$$

orthocentre is $(2,1/2)$

Equation of the line joining $(0,0)$ and

$(2, 1/2)$ is

$$\frac{y-0}{1/2-0} = \frac{x-0}{2-0} \Rightarrow 2y = \frac{x}{2}$$

$$\Rightarrow 4y = x \quad \Rightarrow x - 4y = 0$$

98. If a straight line in XY plane passes through $(-a,-b), (a,b), (k,k), (a^2,a^3)$, for some real numbers a, b and k, where $a \neq 0$, then which of the following options is correct ?

(A) $k = 0$ when $a \neq b$

(B) k is necessarily a positive real number when $a = b$

(C) k is any positive real number when $a \neq b$

(D) $k = a$ or $k = b$ necessarily

(E) $k \neq 0$ when $a \neq b$

Ans:A

$y = \frac{b}{a}x$ Equation of line Passing the given Points (a^2, a^3) satisfy the equation

$$a^3 = \frac{b}{a}a^2$$

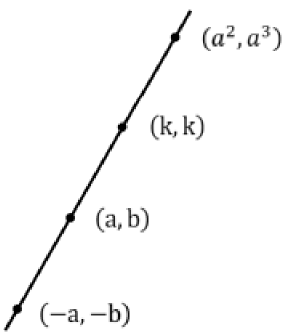
$$\Rightarrow a = b$$

(K,K) obey the equation

$$k = \frac{b}{a}k$$

$$ak - bk = 0$$

$$k(a-b) = 0$$



$k = 0$ when $a \neq b$

99. The line perpendicular to $4x - 5y + 1 = 0$ and passing through the poimt if intersection of the straight lines $x + 2y - 10 = 0$ and $2x + y + 5 = 0$ is

(A) $5x + 4y = 0$ (B) $y + \frac{5}{4}x = \frac{50}{3}$

(C) $5x + 4y = 1$ (D) $y + \frac{5}{4}x = -\frac{50}{3}$

(E) $4x + 5y = 0$

Ans:D

$x + 2y - 10 = 0....(1)$ (1) $\times 2$

$2x + y + 5 = 0....(2)$

$$(1) \times 2 \ \Rightarrow 2x + 4y - 20 = 0....(3)$$

(3)- (2) $\Rightarrow$

$$3y - 25 = 0$$

$$3y = 25$$

$$y = 25/3$$

Substitute $y = \frac{25}{3}$ in (1) we get

$x + 2 \times \frac{25}{3} - 10 = 0$

$x = 10 - \frac{50}{3}$

$= -\frac{20}{3}$

intersecting point $\left(\frac{-20}{3}, \frac{25}{3}\right)$

$$\text{Slope } = \frac{-A}{B} = \frac{-4}{5} = \frac{4}{5}$$

$$\text{perpendicular slop } = \frac{-4}{5}$$

$$y - \frac{25}{3} = \frac{-5}{4}\left(x - \frac{20}{3}\right)$$

$\frac{3y-25}{3} = \frac{-5}{4}\left(\frac{3x+20}{3}\right)$

$12y + 15x = 0$

$y + \frac{5}{4}x = 0$

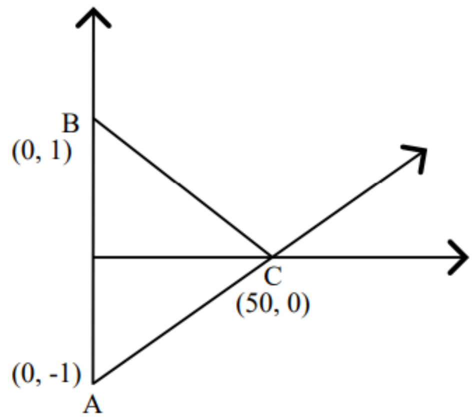
100. A thin particle moves from $(0,1)$ and gets reflected upon hitting the x-axis at $(\sqrt{3}, 0)$. Then the slope of the reflected line is

(A) $\frac{1}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$

(C) $\sqrt{3}$ (D) $-\sqrt{3}$

(E) 0

Ans:A



slop of BC

$m = \frac{y_2-y_1}{x_2-x_1} = \frac{0-1}{\sqrt{3}-0}$

$= \frac{-1}{\sqrt{3}}$

Slope of reflected line is $-m = \frac{1}{\sqrt{3}}$

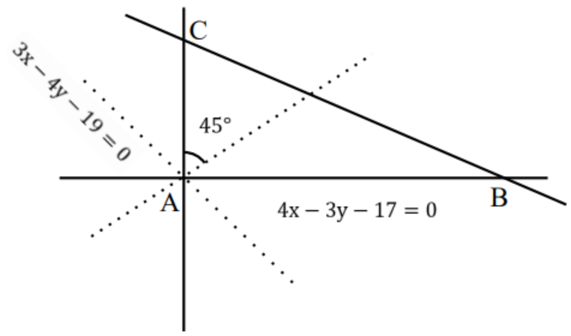
101. If the two sides AB and AC of a triangle are along $4x - 3y - 17 = 0$ and $3x + 4y - 19 = 0$, then the equation of the bisector of the angle between AB and AC is

(A) $x + 7y + 2 = 0$ (B) $7x - y - 36 = 0$

(C) $7x - y + 36 = 0$ (D) $x = y$

(E) $x - 7y + 2 = 0$

Ans:E



Equation of the angle bisectors of two lines

$$A_1x + B_1y + C = 0$$
$$A_2x + B_2y + C = 0$$
$$\frac{A_1x + B_1y + C}{\sqrt{A_1^2 + B_1^2}} = \pm\left(\frac{A_2x + B_2y + C}{\sqrt{A_2^2 + B_2^2}}\right)$$

$$3x + 4y - 19 = \pm(4x - 3y - 17)$$
$$7x + y - 36 = 0 \text{ or}$$
$$x - 7y + 2 = 0$$

102. A point moves in such a way that it remains equidistant from each of the lines $3x \pm 2y = 5$. Then the path along which the point moves is

(A) $x = -\frac{5}{3}$ (B) $y = \frac{5}{3}$

(C) $x = \frac{5}{3}$ (D) $y = -\frac{5}{3}$

(E) $x = 0$

Ans:C

Equation of the angle bisectors of two lines $A_1x+B_1y+C = 0$ and $A_2x+B_2y+C = 0$ is

$$\frac{A_1x + B_1y + C}{\sqrt{A_1^2 + B_1^2}} = \pm\left(\frac{A_2x + B_2y + C}{\sqrt{A_1^2 + B_2^2}}\right)$$

$$\frac{3x + 2y - 5}{\sqrt{13}} = \pm\left(\frac{3x - 2y - 5}{\sqrt{13}}\right)$$

$$3x + 2y - 5 = 3x - 2y - 5$$

or

$$3x + 2y - 5 = -3x + 2y + 5$$
$$6x = 10$$
$$x = \frac{10}{6} = 5/3$$

103. Suppose the line $mx-y+5m-4 = 0$ meets the lines $x+3y+2 = 0, 2x+3y+4 = 0$ and $x-y-5 = 0$ at the points R, S and T, respectively. If R, S and T are at distances r_1 ,r_2 and r_3, respectively, from $(-5,-4)$ and $\left(\frac{15}{r_1}\right)^2 + \left(\frac{10}{r_2}\right)^2 = \left(\frac{6}{r_3}\right)^2$ then the

value of m is

(A) $-\frac{2}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) $-\frac{3}{2}$

(E) 1

Ans:

104. Suppose the point $P(1,1)$ is translated to Q in the direction of $y = 2x$. If $PQ = 1$, then Q is

(A) $(2, 0)$

(B) $(0, 2)$

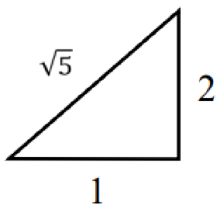
(C) $\left(\frac{\sqrt{2}+1}{\sqrt{2}}, \frac{\sqrt{2}+1}{\sqrt{2}}\right)$

(D) $\left(\frac{\sqrt{5}+1}{\sqrt{5}}, \frac{\sqrt{5}+2}{\sqrt{5}}\right)$

(E) $\left(\frac{2+\sqrt{3}}{2}, \frac{3}{2}\right)$

Ans:D

tan θ = 2



$$\frac{x-1}{\cos\theta} = \frac{y-1}{\sin\theta} = 1$$

$$\frac{x-1}{1/\sqrt{5}} = \frac{y-1}{2/\sqrt{5}} = 1$$

$$x = \frac{1}{\sqrt{5}} + 1 \quad , \quad y = \frac{2}{\sqrt{5}} + 1$$

$$x = \frac{\sqrt{5}+1}{\sqrt{5}} \quad , \quad y = \frac{\sqrt{5}+2}{\sqrt{5}}$$

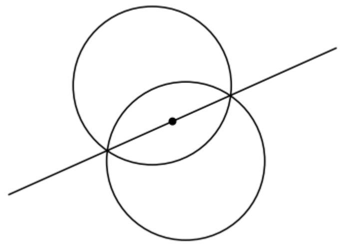
105. Suppose the line joining distinct points P and Q on $(x-2)^2 + (y-1)^2 = r^2$ is the diameter of $(x-1)^2 + (y-3)^2 = 4$. Then the value of r is

(A) 2 (B) 3

(C) 1 (D) 9

(E) 4

Ans:B



$$x^2 + y^2 - 4x - 2y + (5-r)^2 = 0$$

$$x^2 + y^2 - 2x - 6y + 6 = 0$$

$$(1) - (2) \Rightarrow -2x + 4y - r^2 - 1 = 0$$

$$-2 + 12 - r^2 - 1 = 0$$

$$r^2 = 9$$

$$r = 3$$

106. The equation of the circle that can be inscribed in the square formed by $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ is

(A) $x^2 - 8x - 14y + 61 = 0$

(B) $x^2 - 8x - 14y + 71 = 0$

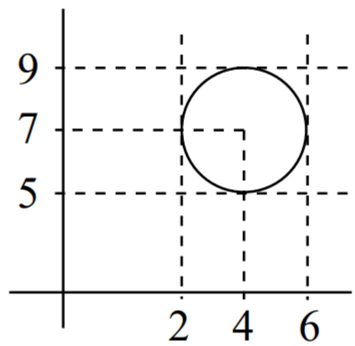
(C) $x^2 - 4x - 7y + 61 = 0$

(D) $x^2 - 4x - 7y + 71 = 0$

(E) $x^2 + 8x + 14y - 61 = 0$

Ans:

$x = 2, 6 \quad y = 5, 9$



$$\text{centre } = (4, 7), r = 2$$

equation

$$(x-4)^2 + (y-1)^2 = 4$$

$$x^2 + y^2 - 8x - 14y + 61 = 0$$

107. For the cirele $C : x^2 + y^2 - 6x + 2y = 0$, which of the following is incorrect

(A) the radius of C is $\sqrt{10}$

(B) $(3, -1)$ lies inside of C

(C) $(7, 3)$ lies outside of C

(D) the line $x + 3y = 0$ intersects C

(E) one of diameters of C is not along $x + 3y = 0$

Ans:E

$$x^2 + y^2 - 6x + 2y = 0$$
$$x^2 - 6x + 9 + y^2 + 2y + 1 = 9 + 1$$
$$(x - 3)^2 + (y + 1)^2 = 10$$

centre $= (3, -1)$ radius $= \sqrt{10}$

(A) is correct

(B)

$(3, -1) \Rightarrow 9 + 1 - 18 - 2$

$= -10 < 0$

$\Rightarrow$ inside

is correct

(c) $(7, 3) \Rightarrow 49 + 9 - 42 + 6 > 0$ outside is correct

$x + 3y = 0; \quad x = -3y.$

$9y^2 + y^2 + 18y + 2y = 0$

(d) $10y^2 + 20y = 0$

$\text{coy}(y + 2) = 0$

$y = 0, \quad y = -2$ is correct

108. For $i = 1, 2, 3, 4$, suppose the points $(\cos\theta_i, \sec\theta_i)$ lie on the boundary of a circle, where $\theta_i \in \left[0, \frac{\pi}{6}\right)$ are distinct. Then $\cos\theta_1 \cos\theta_2 \cos\theta_3 \cos\theta_4$ equals

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) $\frac{1}{8}$ (D) $\frac{1}{16}$

(E) 1

Ans:E

let general point $(\cos\theta, \sec\theta)$ and radius is 1

$$\cos^2\theta + \sec^2\theta = 1$$
$$\cos^2\theta + \frac{1}{\cos^2\theta} = 1$$
$$\cos^2\theta - \cos^2\theta + 1 = 0$$

product roots $\cos\theta_1 \cos\theta_2 \cos\theta_3 \cos\theta_4 = 1/1 = 1$

109. The set of points of the form $(t^2 + t + 1, t^2 - t + 1)$, where t is a real number represents a/an

(A) circle

(B) parabola

(C) ellipse

(D) hyperbola

(E) pair of straight lines

Ans:B

$x = t^2 + t + 1 \quad .....(1)$

$y = t^2 - t + 1 \quad ......(2)$

$(1) - (2) \Rightarrow$

$x - y = 2t$

$t = \frac{x - y}{2}$

substitute t in equation (1)

$$x = \left(\frac{x - y}{2}\right)^2 + \left(\frac{x - y}{2}\right) + 1$$
$$x = \frac{x^2 + y^2 - 2xy}{4} + \frac{x - y}{2} + 1$$
$$x = \frac{x^2 + y^2 - 2xy}{4} + \frac{2x - 2y}{4} + \frac{4}{4}$$
$$4x = x^2 + y^2 - 2xy + 2x - 2y + 4$$
$$2x = x^2 + y^2 - 2xy - 2y + 4$$
$$x^2 + y^2 - 2xy - 2x - 2y + 4 = 0$$

compare with

$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

$a = 1, \quad b = 1, \quad c = 4$

$h = -1, \quad g = -1, \quad f = -1$

$h^2 = ab$, for parabola

$(-1)^2 = 1 \times 1$

$1 = 1$

$\therefore$ Given set of points represent parabola

110. Suppose a and b are the lengths of major and minor axes of an ellipse that passes through the points $(4, 3)$ and $(-1, 4)$. If the major axis of the ellipse lies along the x-axis, then the value of $\frac{1}{a^2} + \frac{16}{b^2}$ is

(A) 4 (B) $\frac{1}{4}$

(C) 2 (D) $\frac{1}{2}$

(E) 1

Ans:B

Equation of the ellipse, $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$
Given, length of major axis, $2A = a$
length of minor axis, $2B = b$

$\frac{x^2}{(a/2)^2} + \frac{y^2}{(b/2)^2} = 1$

Put $(-1, 4)$

$$\frac{1}{\left(\frac{a^2}{4}\right)} + \frac{16}{\frac{b^2}{4}} = 1$$

$$4\frac{1}{a^2} + 4 \times \frac{16}{b^2} = 1$$

$$\therefore \frac{1}{a^2} + \frac{16}{b^2} = \frac{1}{4}$$

111. For a real number t, the equation $(1+t)x^2 + (t-1)y^2 + t^2 - 1 = 0$ represents a hyperbola provided

(A) $|t| < 1$ (B) $|t| > 1$

(C) $|t| = 1$ (D) $t \in (1, \infty]$

(E) $t \in (-\infty, -1]$

Ans:A

$(1+t)x^2 + (t-1)y^2 + t^2 - 1 = 0$

$(1+t)x^2 + (t-1)y^2 = 1 - t^2$

$\frac{(1+t)x^2}{1-t^2} + \frac{(t-1)y^2}{(1-t^2)} = 1$

$\frac{x^2}{(1-t)} - \frac{y^2}{(1+t)} = 1$

$1 - t > 0$, $1 + t > 0$

$1 > t$, $t > -1$

$|t| < 1$

112. Given the points $A(6, -7, 0)$, $B(16, -19, -4)$, $C(0, 3, -6)$ and $D(2, -5, 10)$, the point of intersection of the lines AB and CD is

(A) $(-1, 1, 2)$ (B) $(1, -1, 2)$

(C) $(1, -1, -2)$ (D) $(-1, 1, -2)$

(E) $(1, 1, 2)$

Ans:B

$A(6, -7, 0), B(16, -19, -4), C(0, 3, -6)$

$D(2, -5, 10)$

line AB,

$\frac{x-6}{16-6} = \frac{y+7}{-19+7} = \frac{z-0}{-4-0}$

$\frac{x-6}{10} = \frac{y+7}{-12} = \frac{z}{-4}$....(1)

line CD

$\frac{x-0}{2-0} = \frac{y-3}{-5-3} = \frac{z+6}{10+6}$

$\frac{x}{2} = \frac{y-3}{-8} = \frac{z+6}{16}$....(2)

Substitute each option in equation (1) and (2)

Option $A(-1, 1, 2)$ Equation (1) $\Rightarrow$

$\frac{-1-6}{10} = \frac{1+7}{-12} = \frac{2}{-4}$

$-\frac{7}{10} \neq \frac{8}{-12} = -\frac{1}{2}$ incorrect

option $B(1, -1, 2)$ Equation (1) $\Rightarrow$

$\frac{1-6}{10} = \frac{-1+7}{-12} = \frac{2}{-4}$

$\frac{-5}{10} = \frac{6}{-12} = \frac{2}{-4}$

$-\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2}$ correct

equation (2) $\Rightarrow$

$\frac{1}{2} = \frac{-1-3}{-8} = \frac{2+6}{16}$

$\frac{1}{2} = \frac{-4}{-8} = \frac{8}{16}$

$\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$ correct

$\therefore$ Option (B) is correct

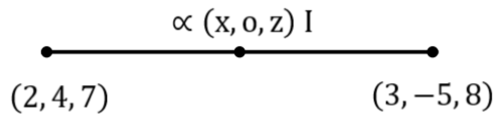
113. If the xz - plane divides the straight line joining the points $(2,4,7)$ and $(3,-5,8)$ in the ratio $\alpha:1$, then the value of α is

(A) $\frac{5}{4}$ (B) $\frac{1}{3}$

(C) $\frac{7}{8}$ (D) $\frac{4}{5}$

(E) $\frac{5}{2}$

Ans:D



$$\frac{-5\alpha+4}{1+\alpha}=0$$

$$-5\alpha+4=0$$

$$-5\alpha=-4$$

$$\therefore \alpha=\frac{-4}{-5}$$

$$\alpha=\frac{4}{5}$$

114. If θ_1, θ_2 and θ_3 are the angles made by a line with the positive directions of the x, y, z axes, then the value of $\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3$ is

(A) -1 (B) 1

(C) 2 (D) -2

(E) 0

Ans:A

$$\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3$$

$$l=\cos\theta_1, m=\cos\theta_2, n=\cos\theta_3$$

$$\therefore l^2+m^2+r^2=1$$

$$\cos^2\theta_1+\cos^2\theta_2+\cos^2\theta_3=1$$

$$\cos 2\theta_1 + \cos 2\theta_2 + \cos 2\theta_3$$

$$=2\cos^2\theta_1-1+2\cos^2\theta_2-1+2\cos^2\theta_3-1$$

$$=2\left[\cos^2\theta_1+\cos^2\theta_2+\cos^2\theta_3\right]-3$$

$$=2\times 1-3$$

$$=-1$$

115. The angle between the lines, whose direction cosines are proportional to $4, \sqrt{3}-1, -\sqrt{3}-1$ and $4, -\sqrt{3}-1, \sqrt{3}-1$, is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

(E) π

Ans:C

Direction cosine of L_1,

$$4, \sqrt{3}-1, -\sqrt{3}-1$$

Direction cosine of L_2,

$$4, -\sqrt{3}-1, \sqrt{3}-1$$

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$\cos\theta=$

$$\frac{4\times4+(\sqrt{3}-1)(-\sqrt{3}-1)+(-\sqrt{3}-1)(\sqrt{3}-1)}{\sqrt{4^2+(\sqrt{3}-1)^2+(-\sqrt{3}-1)^2}\sqrt{4^2+(-\sqrt{3}-1)^2+(\sqrt{3}-1)^2}}$$

$$\frac{16-2-2}{\sqrt{16+3+1-2\sqrt{3}+3+1+2\sqrt{3}}\sqrt{16+3+1+2\sqrt{3}+3+1-2\sqrt{3}}}$$

$$\cos\theta=\frac{12}{\sqrt{24}\sqrt{24}}$$

$$\cos\theta=\frac{12}{24}$$

$$\cos\theta=\frac{1}{2}$$

$$\theta=\frac{\pi}{3}$$

116. Suppose P is the point on the line joining $(-9,4,5)$ and $(11,0,-1)$ that lies closest to the origin O. Then $|OP|^2$ equals to

(A) 3 (B) 4

(C) 2 (D) 9

(E) 1

Ans:D

Equation of line joining $(-9,4,5)$ and $(11,0,-1)$

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$
$$\frac{x+9}{20}=\frac{y-4}{-4}=\frac{z-5}{-6}=\lambda$$
$\therefore x=20\lambda-9, \quad y=-4\lambda+4,$

$z=-6\lambda+5$
$D=|OP|^2$
$=(20\lambda-9)^2+(-4\lambda+4)^2+(-6\lambda+5)^2$
$D=452\lambda^2-452\lambda+122$
Shortest distance, minimum value $\frac{dD}{d\lambda}=0$

$$\frac{dD}{d\lambda}=452\times 2\lambda-452=0$$
$$452\times 2\lambda=452$$
$$2\lambda=1$$
$$\lambda=\frac{1}{2}$$

$$|OP|^2=(20\times\frac{1}{2}-9\Big)^2+\left(-4\times\frac{1}{2}+4\right)^2+\left(-6\times\frac{1}{2}+5\right)^2$$
$$=1+4+4$$
$$=9$$

117. The plane that is perpendicular to the planes $x-y+2z-4=0$ and $2x-2y+z=0$ and passes through $(1,-2,1)$ is
(A) $x+y+1=0$
(B) $2x+y+z-1=0$
(C) $x+y+z=0$
(D) $2x+y-z+1=0$
(E) $x+z-2=0$
Ans:A

The plane is perpendicular to the plane
$x-y+2z-4=0$

$2x-2y+z=0$
normal vector perpendicular to these planes is given by

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ 2 & -2 & 1\end{vmatrix}$$
$=i(-1+4)-j(1-4)+k(-2+2)$
$=3i+3j+0k$
and the plane passes through

$$(1,-2,1)$$

$\therefore$ Equation of plane is,

$$3(x-1)+3(y+2)+0(z-1)=0$$
$$3(x-1)+3(y+2)=0$$
$$3x-3+3y+6=0$$
$$3x+3y+3=0$$
$$\therefore x+y+1=0$$

118. The line of intersection of the planes $3x-6y-2z-15=0$ and $2x+y-2z-5=0$ is
(A) $\frac{x+3}{14}=\frac{y+1}{-2}=\frac{z}{15}$
(B) $\frac{x+3}{-14}=\frac{y+1}{2}=\frac{z}{15}$
(C) $\frac{x-3}{14}=\frac{y+1}{2}=\frac{z}{-15}$
(D) $\frac{x+3}{14}=\frac{y-1}{2}=\frac{z+1}{15}$
(E) $\frac{x-3}{14}=\frac{y+1}{2}=\frac{z}{15}$
Ans:E
* Normal of Plane,
$3x-6y-2z-15=0$ is
$\vec{r_1}=3i-6j-2k$
$*$ Normal of plane $2x+y-2z-5=0$ is,
$\vec{r_2}=2i+j-2k$
line of intersection of two plane is parallel

to $\vec{r_1} \times \vec{r_2}$

$$\vec{r_1} \times \vec{r_2} = \begin{vmatrix} i & j & k \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= i(12+2) - j(-6+4) + k(3+12)$$

$\vec{r_1} \times \vec{r_2}$ =$14i + 2j + 15k$

$$< 14, 2, 15 >$$

Let us assume the point $(a, b, 0)$ on the line,

$3x - 6y - 2z - 15 = 0 \Rightarrow 3a - 6b = 15..(1)$

$2x + y - 2z - 5 = 0 \Rightarrow 2a + b = 5....(2)$

By solving eq(1) and (2)

$a = \frac{45}{15} = 3$

Equation (1) $\Rightarrow$

$3 \times 3 - 6b = 15$

$9 - 15 = 6b$

$-6 = 6b$

$b = -1$

$\therefore$ The point in the line (3,-1,0)

$\therefore$ Equation of line is,

$$\frac{x-3}{14} = \frac{y+1}{2} = \frac{z}{15}$$

119. The plane passing through the points $(2, 1, 0), (5, 0, 1)$ and $(4, 1, 1)$ intersects the x-axis at

(A) $(3, 0, 0)$ (B) $(-3, 0, 0)$

(C) $(0, 0, 0)$ (D) $(1, 0, 0)$

(E) $(-1, 0, 0)$

Ans:A

Plane passing through 3 points,

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x - 2 & y - 1 & z - 0 \\ 5 - 2 & 0 - 1 & 1 - 0 \\ 4 - 2 & 1 - 1 & 1 - 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x - 2 & y - 1 & z \\ 3 & -1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = 0$$

$-1(x-2) - (y-1)(3-2) + z(0+2) = 0$

$-x + 2 - y + 1 + 2z = 0$

$x + y - 2z - 3 = 0$

plane intersect at x-axis, at the point $(x, 0, 0)$

$$x + 0 - 2 \times 0 - 3 = 0$$

$$x = 3$$

$\therefore$ The point is $(3, 0, 0)$

120. Suppose a line parallel to ax + by = 0 (where $b \neq 0$) intersects $5x - y + 4 = 0$ and $3x+4y-4 = 0$, respectively, at P and Q. If the midpoint of PQ is $(1, 5)$, then the value of $\frac{\text{a}}{\text{b}}$ is

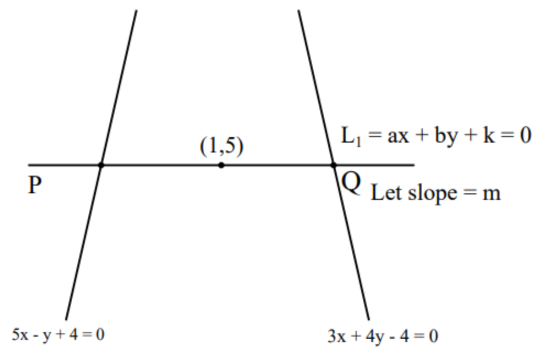
(A) $\frac{107}{3}$ (B) $-\frac{107}{3}$

(C) $\frac{3}{107}$ (D) $-\frac{3}{107}$

(E) 1

Ans:B

Given live parallel to $ax + by = 0$ is $ax + by + k = 0$



$L_1 : \quad y - 5 = m(x - 1)$

$y = mx - m + 5$

$$5x - (mx - m + 5) + 4 = 0$$
$$5x - mx + m - 5 + 4 = 0$$
$$x(5 - m) + (m - 1) = 0$$
x - cordinate of P is,
$$x = \frac{1 - m}{5 - m}$$
$$3x + 4(mx - m + 5) - 4 = 0$$
$$3x + 4mx - 4m + 20 - 4 = 0$$
$$x(3 + 4m) = 4m - 16$$
x - cordinate of Q is,
$$x = \frac{4m - 16}{3 + 4m}$$
Given midpoint of PQ is (1,5)

$$\frac{\frac{1-m}{5-m} + \frac{4m-16}{3+4m}}{2} = 1$$
$$\frac{(1 - m)(3 + 4m) + (4m - 16)(5 - m)}{(5 - m)(3 + 4m)} = 2$$
$$\frac{3 + 4m - 3m - 4m^2 + 20m - 4m^2 - 80 + 16m}{15 + 20m - 3m - 4m^2} = 2$$
$$-8m^2 + 37m - 77 = 30 + 34m - 8m^2$$
$$3m = 107$$
$$\therefore m = \frac{107}{3}$$
Slope of $ax + by + k = 0$
$$m = \frac{-a}{b}$$
$$\frac{107}{3} = \frac{-a}{b}$$
$$\therefore \frac{a}{b} = \frac{-107}{3}$$