XYLEM SAT 2024
JEE SCHOLARSHIP EXAMINATION
SOLUTIONS

PHYSICS

SECTION A

1. (B)

At bottom surface, electric field is zero as $y = 0$

$\therefore$ Electric flux, $\phi_1 = 0$

At top surface, $y = 0.5\mathrm{m}$

$\therefore$ Electric flux, $\phi_2 = \mathrm{EA} = \left(150\mathrm{y}^2\right)(0.5)^2$

$= 150 \times (0.5)^2 \times (0.5)^2$

$= \dfrac{150}{4}(0.5)^2 = \dfrac{150}{16}$

Using Gauss's law $\phi = \dfrac{Q_{\text{in}}}{\epsilon_0} \Rightarrow \dfrac{150}{16} = \dfrac{Q_{\text{in}}}{\epsilon_0}$

$\Rightarrow \mathrm{Q_{in}} = \dfrac{150}{16} \times 8.85 \times 10^{-12} = 8.3 \times 10^{-11}\mathrm{C}$

2. (A)

Electric field in presence of dielectric between the two plates of a parallel plate capacitor is given by,

$\mathrm{E} = \dfrac{\sigma}{\mathrm{K}\varepsilon_0}$

Then, charge density

$\sigma = K\varepsilon_0 E$

$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$

$\approx 6 \times 10^{-7}\mathrm{C/m^2}$

3. (C)

As, we know that $i = neAv_d$

So

$\dfrac{V}{R} = neAv_d$ or $\dfrac{V}{\left(\dfrac{\rho l}{A}\right)} = neAv_d$

$\therefore \quad \rho = \frac{V}{nelv_d} =$ resistivity of the wire

On putting the values we have

$\rho = \dfrac{5}{\left(8 \times 10^{28}\right)\left(1.6 \times 10^{-19}\right)(0.1)\left(2.5 \times 10^{-4}\right)}$

$\approx 1.6 \times 10^{-5}\Omega - \mathrm{m}$

4. (B)

Given

$\mathrm{E_1 = 1\ V, E_2 = 2\ V, E_3 = 3\ V, r_1 = 1\Omega,}$

$\mathrm{r_2} = 1\Omega$ and $\mathrm{r_3} = 1\Omega$

$$\mathrm{V_{AB} = V_{CD}} = \frac{\dfrac{\mathrm{E_1}}{\mathrm{r_1}} + \dfrac{\mathrm{E_2}}{\mathrm{r_2}} + \dfrac{\mathrm{E_3}}{\mathrm{r_3}}}{\dfrac{1}{\mathrm{r_1}} + \dfrac{1}{\mathrm{r_2}} + \dfrac{1}{\mathrm{r_3}}}$$

$$= 1 + \frac{\dfrac{2}{1} + \dfrac{3}{1}}{\dfrac{1}{1} + \dfrac{1}{1} + \dfrac{1}{1}} = \frac{6}{3} = 2\ \mathrm{V}$$

5. (B)

For loop $B = \dfrac{\mu_0 n I}{2a}$

where, a is the radius of loop.

Then, $B_1 = \frac{\mu_0 I}{2a}$

Now, for coil $B = \frac{\mu_0 I}{4\pi} \cdot \frac{2nA}{x^3}$

at the centre x = radius of loop

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2 \times 3 \times (I/3) \times \pi(a/3)^2}{(a/3)^3} = \frac{\mu_0 \cdot 3I}{2a}$$

$$\therefore \quad \frac{B_1}{B_2} = \frac{\mu_0 I/2a}{\mu_0 \cdot 3I/2a}$$

$B_1 : B_2 = 1 : 3$

6. (A)

According to Curie law for paramagnetic substance,

$$\chi \propto \tfrac{1}{T_C} \Rightarrow \frac{\chi_1}{\chi_2} = \frac{T_{C_2}}{T_{C_1}}$$

$$\frac{2.8 \times 10^{-4}}{\chi_2} = \frac{300}{350}$$

$$\chi_2 = \frac{2.8 \times 350 \times 10^{-4}}{300} = 3.266 \times 10^{-4}$$

7. (B)

According to Faraday's law of electromagnetic induction, $e = \frac{-d\phi}{dt}$

$$L \times \frac{di}{dt} = 25 \Rightarrow L \times \frac{15}{1} = 25 \text{ or } L = \frac{5}{3}H$$

Change in the energy of the inductance,

$$\Delta E = \frac{1}{2} L\left(i_1^2 - i_2^2\right) = \frac{1}{2} \times \frac{5}{3} \times \left(25^2 - 10^2\right)$$

$$= \frac{5}{6} \times 525 = 437.5 \text{ J}$$

8. (D)

As $V(t) = 220 \sin 100\pi t$

so, $I(t) = \frac{220}{50} \sin 100\pi t$ i.e.,

$I = I_m = \sin(100\pi t)$

For $I = I_m$

$$t_1 = \frac{\pi}{2} \times \frac{1}{100\pi} = \frac{1}{200} \text{sec.}$$

and for $I = \frac{I_m}{2}$

$$\Rightarrow \frac{I_m}{2} = I_m \sin(100\pi t_2) \Rightarrow \frac{\pi}{6} = 100\pi t_2 \Rightarrow t_2$$

$$= \frac{1}{600} \text{ s}$$

$$\therefore t_{req} = \frac{1}{200} - \frac{1}{600} = \frac{2}{600} = \frac{1}{300} \text{ s} = 3.3 \text{ ms}$$

9. (B)

$B = B_0 \sin(\omega t \pm 16x)$

$$B_0 = \frac{E_0}{C} = \frac{60}{3 \times 10^8} = 2 \times 10^{-7}$$

$$\omega = 2\pi f = 2\pi \times 23 - 9 \times 10^9 = 1.5 \times 10^{11}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\frac{v}{v}} = \frac{2\pi\nu}{v}$$

$$= \frac{2 \times 3 - 14 \times 23 - 9 \times 10^9}{3 \times 10^8} = 0.5 \times 10^3$$

$$B = 2 \times 10^{-7} \sin\left(0.5 \times 10^3 z - 1.5 \times 10^{11} t\right) \hat{\imath}$$

10. (C)

$$+5 = -\frac{v}{u} \Rightarrow v = -5u$$

Using mirrors formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{-5u} + \frac{1}{u} = \frac{-1}{0.4}$$

$\therefore u = -0.32$ m

11. (C)

For minimum deviation:

$$r_1 = r_2 = \frac{A}{2} = 30^\circ$$

by Snell's law $\mu_1 \sin i = \mu_2 \sin r$

$$1 \times \sin i = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2} \Rightarrow i = 60$$

12. (B)

$$\frac{I_{max}}{I_{min}} = 16$$

$$\Rightarrow \frac{A_{max}}{A_{min}} = 4$$

$$\Rightarrow \frac{A_1 + A_2}{A_1 - A_2} = \frac{4}{1}$$

Using componendo & diviendo.

$$\frac{A_1}{A_2} = \frac{5}{3} \Rightarrow \frac{I_1}{I_3} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

13. (B)

Given :
$d = 0.5$ mm $= 0.5 \times 10^{-3}$ m and $D = 0.5$ m

Fringe width $\beta = \frac{\lambda D}{d}$

$$= \frac{5890 \times 10^{-10} \times 0.5}{0.5 \times 10^{-3}} = 589 \times 10^{-6} \text{ m}$$

Hence, distance between the first and third bright fringe

$= 2\beta$

$= 2 \times 589 \times 10^{-6}$ m

$= 1178 \times 10^{-6}$ m

14. (C)

de Broglie wavelength (λ) is given by
$K = qV$

$$\hat{A} = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2\,m\mathbf{V}}} (\because p = \sqrt{2mK})$$

Substituting the values we get

$$\therefore \frac{\lambda_A}{\lambda_B} = \frac{\sqrt{2\, m_B q_B V_B}}{\sqrt{2\, m_A q_A V_A}} = \sqrt{\frac{4\text{ m} \cdot \text{q} \cdot 2500}{\text{m} \cdot \text{q} \cdot 50}}$$

$= 2\sqrt{50} = 2 \times 7.07 = 14.14$

15. (B)

$$\frac{1}{\lambda_1} = -R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16} \quad \therefore \frac{\lambda_2}{\lambda_1} = \frac{80}{108}$$

$$\lambda_2 = \frac{80}{108}\lambda_1 = \frac{80}{108} \times 660 = 488.9 \text{ nm.}$$

16. (C)

$$\frac{V_1}{V_2} = \frac{8}{27}$$

$m_1\ V_1 = m_2\ V_2$

$$\frac{m_1}{m_2} = \frac{V_2}{V_1} = \frac{27}{8}$$

$$\frac{\rho \times \frac{4}{3}\pi R_1^3}{\rho \times \frac{4}{3}\pi R_2^3}$$

$$\left(\frac{R_1}{R_2}\right) = \left(\frac{27}{8}\right)^{\frac{1}{3}}$$

$$= \left(\frac{3}{2}\right)^{3 \times \frac{1}{3}}$$

$$\frac{R_1}{R_2} = \frac{3}{2}$$

17. (C)

For different values of A& B output y is as follows

A	B	P	Q	Y
0	0	0	0	1
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

Output y $= \overline{(A \cdot B) \cdot A + B}$ matches with option (C).

18. (D)

3.1eV

For photodiode to detect hc/ $\lambda >$ band gap energy $\Rightarrow$ band gap energy max hc/ λ_{max}

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}}$$

$= 5 \times 10^{-19}$ J

$= 3.1$eV

19. (C)

Energy is released when stability increases. This will happen when binding energy per nucleon increases i.e.,

$$\left(\frac{B.E.}{A}\right)_{\text{Product}} > \left(\frac{B.E.}{A}\right)_{\text{Reactant}}.$$

$\because \quad |M_1\ V_1| = |M_2\ V_2|$

Reactant	**Product**
Reaction (a) $60 \times 8.5\text{MeV} = 510\text{MeV}$	$2 \times 30 \times 5 = 300\text{MeV}$
Reaction (b) $120 \times 7.5 = 900\text{MeV}$	$(90 \times 8 + 30 \times 5) = 870\text{MeV}$
Reaction (c) $120 \times 7.5 = 900\text{MeV}$	$2 \times 60 \times 8.5 = 1020\text{MeV}$
Reaction (d) $90 \times 8 = 720\text{MeV}$	$(60 \times 8.5 + 30 \times 5) = 600\text{MeV}$

20. (D)

SECTION B

21. (36)

When two spheres charges $q_1' = 2.1\text{nC}$ and $q_2' = -0.1\text{nC}$ are brought into contact and then separated by a distance $r = 0.5$ m then,

$q_1' = q_2' = \dfrac{Q_1 + Q_2}{2} = 1\text{nC}$

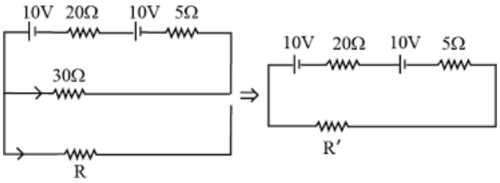
Electrostatic force between the two charged sphere,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1' q_2'}{r^2} = 9 \times 10^9 \times \frac{10^{-9} \times 10^{-9}}{(0.5)^2}$$

$= 36 \times 10^{-9}$ N

$\therefore x = 36.$

22. (30)



The resistance of 30Ω is in parallel with R. Their effective resistance

$$\frac{1}{R'} = \frac{1}{30} + \frac{1}{R}$$

$$R' = \frac{30R}{30 + R}$$

Also, $V = IR \Rightarrow 10 = \dfrac{20 \times 20}{R' + 25}$

$\Rightarrow R' + 25 = 40 \Rightarrow R' = 15$

$$R' = 15 = \frac{30R}{30 + R}$$

$\Rightarrow 30 + R = 2R \Rightarrow R = 30\Omega$

23. (30)

Refraction formula (refraction from curved surface)

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \left[\because \lambda \times \frac{1}{\mu}\right]$$

$\mu_1 = 1$ (in air) $\mu_2 = 1.5$ (inside the surface)

$$\frac{1.5}{+10} - \frac{1}{-15} = \frac{1.5 - 1}{+R}$$

$$\Rightarrow \frac{1.5}{10} + \frac{1}{15} = \frac{0.5}{R} \Rightarrow R = \frac{30}{13} \text{ m}$$

$\therefore x = 30$

24. (3)

For current leads the voltage by $45°$

$$\tan 45° = \frac{x_C - x_L}{R}$$

$\Rightarrow x_C - x_L = R$

or, $\dfrac{1}{\omega C} - \omega L = R$

or, $\dfrac{1}{\omega C} - 300 \times 0.03 = 1$

$$\frac{1}{\omega C} = 10 \Rightarrow C = \frac{1}{10\omega} = \frac{1}{10 \times 300}$$

$\therefore \quad C = \dfrac{1}{3} \times 10^{-3}$

Hence, value of $x = 3$.

25. (5)

For a doped semi-conductor in thermal equilibrium $n_e n_h = n_i^2$

$$\Rightarrow n_e = \frac{n_i^2}{n_h} = \frac{\left(1.5 \times 10^{16}\right)^2}{4.5 \times 10^{22}}$$

$$= \frac{1.5 \times 1.5 \times 10^{32}}{4.5 \times 10^{22}} = 5 \times 10^9 \text{ m}^{-3}$$

CHEMISTRY

SECTION A

26. Ans: (D)

Movement of solvent molecules takes place from low concentrated solution (mango) to more concentrated solution (salt solution). This process of movement of solvent molecules is called osmosis and due to osmosis, a raw mango shrinks.

27. Ans: (D)

$\Delta T_f = iK_f m$

m is same for all the solutions thus, $\Delta T_f \propto i$

$[Co(H_2O)_6]Cl_3 \Rightarrow 4$ ions $(i = 4)$

$[Co(H_2O)_5Cl]Cl_2 \cdot H_2O \Rightarrow 3$ ions $(i = 3)$

$[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O \Rightarrow 2$ ions $(i = 2)$

$[Co(H_2O)_3Cl_3] \cdot 3H_2O \Rightarrow$ No ion $(i = 1)$

Freezing point of solution increases, the value of i decreases. So, highest freezing point will be of $[Co(H_2O)_3Cl_3] \cdot 3H_2O$ solution.

28. Ans: (B)

During electrolysis of dilute sulphuric acid, the following reaction takes place at anode :

$2H_2O_{(l)} \rightarrow O_{2(g)} + 4H^+_{(aq)} + 4e^-; E^\circ_{cell}$

$= +1.23$ V

i.e., $O_{2(g)}$ will be liberated at anode.

29. Ans: (B)

The acidic strength decreases in the order:

$$\underset{(A)}{HCOOH} > \underset{(C)}{C_6H_5COOH} > \underset{(B)}{CH_3COOH}.$$

As the acidic strength decreases, rate of dissociation decreases and hence conductivity decreases.

30. Ans: (A)

$t_{1/2} = 100$ second(50% reaction)

After 200 seconds, 75% of reaction will be completed, i.e., $t_{75\%} = 200$ seconds.

Thus, it follows first order kinetics as half-life is independent of concentration and follows the relation, $t_{3/4} = 2 \times t_{1/2}$

31. Ans: (C)

32. Ans: (C)

NH_3 has abnormally high boiling points because of their tendency to form hydrogen bonds. The order of boiling point will be $SbH_3 > NH_3 > AsH_3 > PH_3$

33. Ans: (A)

34. Ans: (D)

The atomic and ionic radii of Zr and Hf are almost identical due to poor shielding effect of $4f$-electrons, which lead to the lanthanoid contraction.

35. Ans: (A)

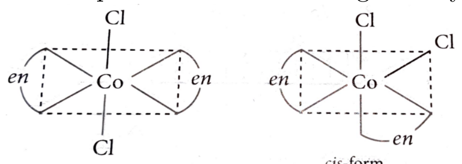
Mn^{2+} : $3d^5$, no. of unpaired electrons $(n) = 5$ Magnetic moment $(\mu) = \sqrt{n(n+2)}$ B.M.

$= \sqrt{5(5+2)}$ B.M. $= \sqrt{35} = 5.92$ B.M.

36. Ans: (A)

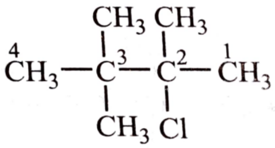
37. Ans: (A)

$[CoCl_2(en)_2]$, exhibit geometrical isomerism, as the coordination number of Co is 6 and this compound has octahedral geometry.



trans-form (optically inactive) *cis*-form (optically active)

38. Ans: (D)



2-Chloro-2, 3, 3-trimethylbutane

39. Ans: (C)

Tertiary alcohols are most reactive and immediately produce turbidity at room temperature while primary alcohols do not react with Lucas reagent at room temperature.

40. Ans: (C)

41. Ans: (B)

The carbonyl carbon is sp^2 hybridised and forms three sigma bonds. Thus carbonyl carbon and three atoms attached to it lie in the same plane and π-electron cloud is above and below this plane.

42. Ans: (C)

43. Ans: (C)

Only $-CH_3$ group is electron donating group. Hence, it increases the electron density on nitrogen making it most basic.

44. Ans: (B)

Glycosidic linkage between $C1$ of α-glucose and $C2$ of β-fructose is found in sucrose.

45. Ans: (B)

In secondary structure of RNA, helices are present which are only single stranded.

SECTION B

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46. Ans: (6)

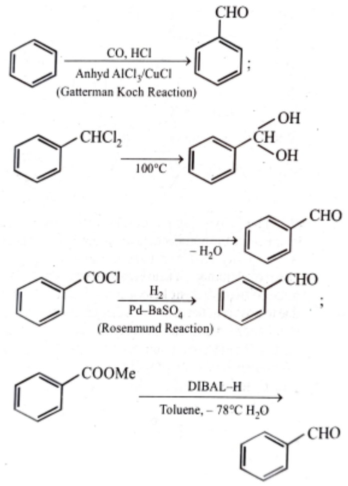
The oxidation state of Cr changes

from +6 to +3 .

$$\underset{1\ \text{mol}}{+6\ Cr_2O_7^{-2}} + \underset{6\ \text{mol}}{6e^-} \longrightarrow Cr^{+3}$$

$\Rightarrow$ number of faradays = moles of electrons = 6

47. Ans: (4)



48. Ans: (3)

Magnetic moment $\mu = \sqrt{n(n+2)}$BM

$1.73 = \sqrt{n(n+2)}$ $\therefore$ $n = 1$, it has one unpaired electron hence electronic configuration is $[Ar]3d^1$ and electronic configuration for Z = 22 is $[Ar]3d^2 4s^2$. Hence charge on Ti is +3.

49. Ans: (0)

$\overset{+6}{Cr}O\ O_4^{2-}, \overset{+6}{Cr_2}O_7^{2-}$
Hence, the difference is zero.

50. Ans: (5)

Mischmetal is an alloy which contains rare earth elements (94-95%), iron (5%) and traces of sulphur, carbon, silicon, calcium and aluminium. It is used in gas lighters, tracer bullets and shells.

MATHEMATICS

SECTION A

51. (C)

We have, $f(x) = x^2 + 9$

The graph of $f(x)$ is same as graph of x^2 shifted up by 9 units.

We know that range of $g(x) = x^2$ is $[0, \infty)$.

$\therefore$ Range of $f(x) = x^2 + 9$ is $[9, \infty)$.

52. (A)

The given function is $f(x) = \dfrac{\log_3(x+7)}{x^2 - 5x + 6}$

$f(x)$ is defined if $x + 7 > 0 \Rightarrow x > -7$

and $x^2 - 5x + 6 = (x-3)(x-2) \neq 0$

$\therefore$ Domain of $f(x) = (-7, \infty) - \{2, 3\}$.

53. (D)

Consider $\cos^{-1}\left(\cos\left(\frac{7\pi}{4}\right)\right)$

$= \cos^{-1}\left(\cos\left(2\pi - \frac{\pi}{4}\right)\right)$

$= \cos^{-1}\left(\cos\frac{\pi}{4}\right)$

$[\because \cos(2\pi - \theta) = \cos\theta]$

$= \frac{\pi}{4}$

54. (B)

$$\begin{bmatrix} 3x-y & x+3y \\ 2x-z & 2y+z \end{bmatrix} = \begin{bmatrix} 7 & 9 \\ 5 & 5 \end{bmatrix}$$

$3x - y = 7 \quad -(i)$

$x + 3y = 9$

$3x + 9y = 27 - (ii)$

(ii)-(i) gives

$10y = 20$

$y = 2$

$x = 3$

Similarly

$2x - z = 5$

$2y + z = 5$

$\Rightarrow \quad z = 1$ and $x = 3$

$x + y + z = 3 + 2 + 1 = 6$

55. (D)

$$\begin{vmatrix} 3 & 1 & -1 \\ 1 & 0 & -1 \\ 2 & 2 & a \end{vmatrix} \neq 0 \Rightarrow 3(2) - 1(a+2) - 1(2) \neq 0$$

$\Rightarrow 6 - a - 2 - 2 \Rightarrow 0 \Rightarrow a \neq 2$

56. (C)

We have, $|(4-x)\vec{a}| < |3\vec{a}|$, where $\vec{a}$ is a non-zero vector and $x \in \mathbb{R}$

On squaring both sides, we get

$$|(4-x)\vec{a}|^2 < |3\vec{a}|^2$$
$$\Rightarrow \quad (4-x)^2 < 9 \Rightarrow (4-x) < 3 \text{ or }$$
$$(4-x) > -3$$
$$\Rightarrow 4-3 < x \text{ or } -x > -7$$
$$\Rightarrow 1 < x \text{ or } x < 7 \Rightarrow 1 < x < 7$$

57. (C)

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$
$$\Rightarrow \lim_{x\to 2^-} ax+3 = \lim_{x\to 2^+} a^2x-1$$
$$\Rightarrow \quad 2a+3 = 2a^2-1 \Rightarrow 2a^2-2a-4=0$$
$$\Rightarrow \quad a^2-a-2=0 \Rightarrow (a+1)(a-2)=0$$
$$\Rightarrow a=-1 \text{ and } 2$$

58. (B)

We have, $2^x + 2^y = 2^{x+y}$ Differentiating w.r.t. x, we get

$$2^x \cdot \log 2 + 2^y \cdot \log 2 \cdot \frac{dy}{dx} = 2^{x+y} \cdot \log 2 \left(1 + \frac{dy}{dx}\right)$$
$$\Rightarrow \frac{dy}{dx}\left(2^y - 2^{x+y}\right) = 2^{x+y} - 2^x$$
$$\Rightarrow \frac{dy}{dx} = \frac{2^{x+y}-2^x}{2^y - 2^{x+y}}$$
$$\therefore \frac{dy}{dx}_{\text{at }(1,1)} = \frac{2^2-2}{2-2^2} = -1$$

59. (A)

Given, $f'(x) = g(x+1)$

$$\Rightarrow \quad f''(x) = g'(x+1). \text{ Also, } g'(x) = h(x-1)$$
$$\Rightarrow \quad g'(x+1) = h(x) \Rightarrow f''(x) = h(x)$$
$$\Rightarrow f''(2x) = h(2x)$$

60. (D)

$$f(x) = x^5 e^{-x}$$
$$\Rightarrow f'(x) = -x^5e^{-x} + e^{-x}\cdot 5x^4 = e^{-x}x^4(5-x)$$

For increasing, $f'(x) > 0$

$$\Rightarrow e^{-x}x^4(5-x) > 0 \Rightarrow x < 5 \Rightarrow x \in (-\infty, 5)$$

61. (C)

$$\text{Let } I = \int \frac{1}{\cos x(\sin x + 2\cos x)} dx$$
$$= \int \frac{1}{\cos x \sin x + 2\cos^2 x} dx$$
$$= \int \frac{1}{\cos^2 x(\tan x + 2)} dx$$
$$= \int \frac{\sec^2 x}{(\tan x + 2)} dx \qquad \ldots (i)$$

Put $\tan x + 2 = t \Rightarrow \sec^2 x dx = dt$

$$\therefore \quad \text{(i) becomes, } I = \int \frac{dt}{t} = \log|t| + C$$
$$= \log|\tan x + 2| + C$$

62. (D)

$$\text{Let } I = \int \frac{2x + \sin 2x}{1+\cos 2x} dx$$
$$= 2\int \frac{x}{1+\cos 2x} dx + \int \frac{\sin 2x}{1+\cos 2x} dx$$
$$= 2\int \frac{x}{2\cos^2 x} dx + \int \frac{2\sin x\cos x}{2\cos^2 x} dx$$
$$= \int \frac{I}{x} \sec^2 x dx + \int \tan x dx$$
$$= x\int \sec^2 x dx - \int \left(1 \cdot \int \sec^2 x dx\right) dx + \int \tan x dx$$
$$= x\tan x - \int \tan x dx + \int \tan x dx + C$$
$$= x\tan x + C$$

63. (C)

On squaring the equation, we get

$$\left[\frac{2d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]^3=\left(\frac{d^3y}{dx^3}\right)^2$$

$\therefore \quad$ Order $=3,$ degree $=2$

64. (B)

$\frac{dy}{dx}=e^x+1 \Rightarrow dy=(e^x+1)\,dx \Rightarrow y=e^x+x+C$

65. (C)

Total number of outcomes = 8 i.e., {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

Number of favourable outcomes = 4

i.e., { TTH, THT, HTT, TTT } = 4

$\therefore P($ getting at least 2 tails $)=\frac{4}{8}=\frac{1}{2}$

66. (D)

E and F are independent, therefore

$P(E\cap F)=P(E)\cdot P(F)=\frac{1}{4}\cdot a=\frac{a}{4}$

Now, $P(E\cup F)=P(E)+P(F)-P(E\cap F)$

$\Rightarrow \frac{2}{5}=\frac{1}{4}+a-\frac{a}{4} \Rightarrow \frac{2}{5}-\frac{1}{4}=\frac{3a}{4}$

$\frac{3}{20}=\frac{3a}{4} \Rightarrow a=\frac{1}{5}$

67. (B)

Given, $\vec{a}=-2\hat{i}+\hat{j}-\hat{k}$

The direction cosines of $\vec{a}$ are

$$\left(\frac{-2}{\sqrt{(-2)^2+(1)^2+(-1)^2}},\frac{1}{\sqrt{(-2)^2+(1)^2+(-1)^2}}\frac{-1}{\sqrt{(-2)^2+(1)^2+(-1)^2}}\right)$$

$$=\left(\frac{-2}{\sqrt{4+1+1}},\frac{1}{\sqrt{4+1+1}},\frac{-1}{\sqrt{4+1+1}}\right)$$

$$=\left(\frac{-2}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{-1}{\sqrt{6}}\right)$$

68. (D)

$\cos\theta=\frac{1\cdot 3+2\cdot 2+3\cdot 1}{\sqrt{1^2+2^2+3^2}\sqrt{3^2+2^2+1^2}}$

$\Rightarrow \cos\theta=\frac{10}{\sqrt{14}\sqrt{14}}=\frac{10}{14}=\frac{5}{7}$

69. (D)

Let $\left(\frac{1}{2},\frac{1}{2}\right)\notin R$, because $\frac{1}{2}>\left(\frac{1}{2}\right)^2=\frac{1}{4}$

$\therefore$ R is not reflexive.

Now, let $(1,4)\in R$ as $1\leq 4^2$

But 4 is not less than 1^2

$\therefore (4,1)\notin$ R

$\therefore$ R is not symmetric

Next,
consider $(3,2),(2,1.5)\in R($ as $3<2^2$
&$2<1.5^2$=2.25

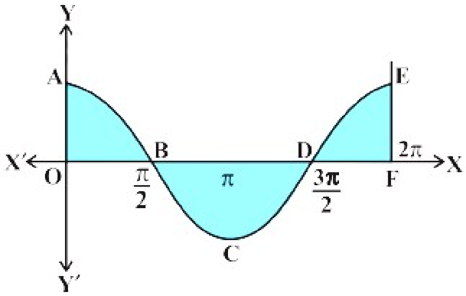
But $3>1.5^2=2.25$

$\therefore (3,1.5)\notin$ R

$\therefore$ R is not transitive

Thus R is neither reflexive, symmetric, nor transitive

70. (C)



From the Figure the required area = area of the region OABO+ area of the region BCDB+ area of the region DEFD.

Thus, we have the required area

$$= \int_0^{\frac{\pi}{2}} \cos x dx + \left| \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x dx \right| + \int_{\frac{3\pi}{2}}^{2\pi} \cos x dx$$

$$= [\sin x]0^{\frac{\pi}{2}} + \left| [\sin x] \frac{\pi}{2}^{\frac{3\pi}{2}} \right| + [\sin x]_{\frac{3\pi}{2}}^{2\pi}$$

$$= 1 + 2 + 1 = 4$$

SECTION B

71. (2)

We have, $f\left(\frac{x+1}{2x-1}\right) = 2x$

Put $x = 1$ on both sides, we get $f(2) = 2$

72. (8)

Let $\tan^{-1}\left(\frac{3}{4}\right) = \theta$

$\Rightarrow \tan\theta = \frac{3}{4}$

$\therefore \quad \cos\theta = \frac{4}{5}$

$\Rightarrow \alpha = \frac{4}{5}$

$10\alpha = 8$

73. (3)

Let $B = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

adj $B = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$,

where C_{ij} are cofactors.

$$= \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = A$$

$\therefore \quad a_1 = 7, b_2 = 1, c_3 = 1$

$\therefore \quad a_1 = 7, b_2 = 1, c_3 = 1$ and $b_1 = -3$ and $a_2 = -1$

$$\therefore \quad \frac{a_1 + b_2 + c_3}{b_1 a_2} = \frac{7+1+1}{3} = \frac{9}{3} = 3$$

74. (10)

We have $y = f\left(x^2 + 2\right)$ ...(i)

Differentiating (i) w.r.t. x, we get

$\frac{dy}{dx} = f'\left(x^2 + 2\right)(2x)$

$\Rightarrow \left.\frac{dy}{dx}\right|_{\text{at } x=1} = 2f'(3)$=2 × 5=10

75. (5)

We have, $\int\limits_{-1}^{4} f(x)dx = 4$

$\Rightarrow \int\limits_{-1}^{2} f(x)dx + \int\limits_{2}^{4} f(x)dx = 4$ ...(i)

Also, $\int\limits_{2}^{4} (3 - f(x))dx = 7 \Rightarrow \int_2^4 3dx - \int\limits_{2}^{4} f(x)dx = 7$

$\Rightarrow \quad 3[x]_2^4 - 7 = \int_2^4 f(x)dx \Rightarrow \int_2^4 f(x)dx = -1$ ...(ii)

From (i) and (ii), we have $\int_{-1}^{2} f(x)dx = 4 + 1 = 5$